Discussiones Mathematicae
General Algebra and Applications 41 (2021) 321–342
https://doi.org/10.7151/dmgaa.1363

UNI-SOFT QUASI-HYPERIDEALS OF ORDERED SEMIHYPERGROUPS

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Abstract

The main purpose of this article is to study ordered semihypergroups in the context of uni-soft quasi-hyperideals. In this article, using the notion of soft-union sets in ordered semihypergroups, we introduce the concept of union-soft (uni-soft) quasi-hyperideal and the related properties are investigated. We prove that every uni-soft left (right) hyperideal is a uni-soft quasi-hyperideal but the converse is not true which is shown with help of an example. We present the characterizations of left (right) simple and completely regular ordered semihypergroups in terms of uni-soft quasi-hyperideals. Furthermore we define semiprime uni-soft quasi-hyperideal and characterize completely regular ordered semihypergroup using this notion.

Keywords: uni-soft bi-hyperideal, uni-soft quasi-hyperideal, semiprime uni-soft quasi-hyperideal, left (right) simple, regular and completely regular ordered semihypergroup.

2010 Mathematics Subject Classification: 20N20.

1. Introduction

The concept of hyperstructure was first introduced by Marty [18], at the eighth Congress of Scandinavian Mathematicians in 1934, when he defined hypergroups and started to analyze its properties. The core cause which attracts researches towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Thus algebraic hyperstructures are natural extension of classical algebraic structures. Now, the theory of algebraic hyperstructures has become a well-established branch in algebraic theory and it has extensive applications in many branches of mathematics and applied science. In a recent monograph [2], Corsini and Leoreanu have presented numerous applications of algebraic hyperstructures. Later on, people have developed the semihypergroups, which are the simplest algebraic hyperstructures having closure and associative properties. A comprehensive review of the theory of hyperstructures can be found in [1–5, 10–12, 20–22, 24, 25].

The real world is too complex for our immediate and direct understanding. We create "models" of reality that are simplifications of aspects of the real word. Unfortunately these mathematical models are too complicated and we cannot find the exact solutions. The uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical sciences and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world or to the limitations of the means used to measure objects. The classical set theory, which is based on the crisp and exact case may not be fully suitable for handling such problems of uncertainty. To overcome these difficulties, Molodtsov [19], introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties. Molodtsov soft set theory is a kind of new mathematical model for coping with uncertainty from a parameterization point of view, in soft set theory, the problem of setting membership function does not arise, which makes the theory easily applied to several fields. Worldwide, there has been a rapid growth in interest in soft set theory and its applications in recent years, (see [7–9, 13, 14, 16, 17, 26]).

In this paper, we introduce the notion of uni-soft quasi-hyperideals of ordered semihypergroups. We show that every uni-soft quasi-hyperideal is a uni-soft bi-hyperideal and in a regular ordered semihypergroup, uni-soft quasi-hyperideals and uni-soft bi-hyperideals coincide. We characterized ordered semihypergroups in terms of uni-soft quasi-hyperideals. We present the characterizations of left (right) simple and completely regular ordered semihypergroups. We define semi-prime uni-soft quasi-hyperideal and characterized completely regular ordered semihypergroup using this notion.

2. Preliminaries

2.1. Basic results on ordered semihypergroups

A hypergroupoid is a nonempty set S equipped with hyperoperation $\circ$, that is a map $\circ : S \times S \longrightarrow P^*(S)$, where $P^*(S)$ denotes the set of all nonempty subsets of S (see [18]). We shall denote by $x \circ y$, the hyperproduct of elements x, y of S. A hypergroupoid $(S, \circ)$ is called a *semihypergroup* if $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in S$. Let A, B be the nonempty subsets of S. Then the hyperproduct of A and B is defined as $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$. We shall write $A \circ x$ instead of $A \circ \{x\}$ and $x \circ A$ for $\{x\} \circ A$.

Definition 1 (see [22]). An algebraic hyperstructure $(S, \circ, \leq)$ is called an ordered semihypergroup (also called po-semihypergroup) if $(S, \circ)$ is a semihypergroup and $(S, \leq)$ is a partially ordered set such that the monotone condition holds as follows.

$a \leq b$ implies that $x \circ a \leq x \circ b$ and $a \circ x \leq b \circ x$ for all $x, a, b \in S$, where, if $A, B \in P^*(S)$,then we say that $A \preceq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$. If $A = \{a\}$ then we write $a \preceq B$ instead of $\{a\} \preceq B$.

Definition 2 (see [1]). A nonempty subset A of an ordered semihypergroup $(S, \circ, \leq)$ is called a *subsemihypergroup* of S if for all $x, y \in A$ implies that $x \circ y \subseteq A$.

Equivalently. A nonempty subset A of an ordered semihypergroup $(S, \circ, \leq)$ is called a subsemihypergroup of S if $A \circ A \subseteq A$.

Definition 3 (see [22]). Let $(S, \circ, \leq)$ be an ordered semihypergroup and A be a nonempty subset of S. Then A is called a *left* (*right*) *hyperideal* of S if:

(1) $S \circ A \subseteq A$ $((A \circ S) \subseteq A)$.

(2) If $a \in A$ and $S \ni b \leq a$ then $b \in A$.

If A is both a right hyperideal and a left hyperideal of S, then it is called a hyperideal of S.

Definition 4 (see [22])**.** A subsemihypergroup A of an ordered semihypergroup $(S, \circ, \leq)$ is called a bi-hyperideal of S if:

(1) $A \circ S \circ A \subseteq A$.

(2) If $a \in A$ and $S \ni b \leq a$ then $b \in A$.

For $A \subseteq S$, we denote $(A] = \{t \in S \mid t \leq h$ for some $h \in A\}$.

Definition 5 (see [1])**.** A nonempty subset Q of an ordered semihypergroup $(S, \circ, \leq)$ is called a quasi-hyperideal of S if:

(1) $(Q \circ S] \cap (S \circ Q] \subseteq Q$.

(2) If $a \in Q$ and $S \ni b \leq a$ then $b \in Q$.

The quasi-hyperideal of S generated by a $(a \in S)$ is denoted by $Q(a)$ and is defined as $Q(a) = (a \cup ((a \circ S] \cap (S \circ a])]$.

Lemma 1 (see [16])**.** *Let $(S, \circ, \leq)$ be an ordered semihypergroup and A, B are the nonempty subsets of S. Then the following statements hold:*

(1) $A \subseteq (A]$.

(2) $A \subseteq B$ *implies that* $(A] \subseteq (B]$.

(3) $(A] \circ (B] \subseteq (A \circ B]$.

(4) $((A] \circ (B]] = (A \circ B]$.

(5) $((A]] = (A]$.

For the sake of simplicity throughout this paper, we denote by $a^n = a \circ a \circ \cdots \circ a$ n-copies.

Definition 6 (see [22])**.** (1) An ordered semihypergroup $(S, \circ, \leq)$ is called left (resp., right) *regular* if for each $a \in S$ there exists $x \in S$ such that $a \leq x \circ a \circ a$ (resp., $a \leq a \circ a \circ x$).

Equivalently. An ordered semihypergroup $(S, \circ, \leq)$ is called left (resp., right) *regular* if for each $a \in S$, $a \in \left(S \circ a^2\right]$ (resp., $a \in \left(a^2 \circ S\right]$).

(2) An ordered semihypergroup $(S, \circ, \leq)$ is called *regular* if for each $a \in S$ there exists $x \in S$ such that $a \leq a \circ x \circ a$.

2.2. Basic concepts of soft sets

In what follows, we take $E = S$ as the set of parameters, which is an ordered semihypergroup, unless otherwise specified.

From now on, U is an initial universe set, E is a set of parameters, $P(U)$ is the power set of U and $A, B, C \cdots \subseteq E$.

Definition 7 (see [19]). A *soft set* f_A over U is defined as $f_A : E \longrightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Hence f_A is also called an *approximation function.*

A soft set f_A over U can be represented by the set of ordered pairs $f_A = \{(x, f_A(x)) | x \in E, f_A(x) \in P(U)\}$. It is clear from Definition that a soft set is a *parameterized family* of subsets of U. Note that the set of all soft sets over U will be denoted $S(U)$.

Definition 8 (see [19]). (i) Let $f_A, f_B \in S(U)$. Then f_A is called a *soft subset* of f_B, denoted by $f_A \widetilde{\subseteq} f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Two soft sets f_A and f_B are said to be equal soft sets if $f_A \widetilde{\subseteq} f_B$ and $f_B \widetilde{\subseteq} f_A$ and is denoted by $f_A \widetilde{=} f_B$.

(ii) Let $f_A, f_B \in S(U)$. Then the *soft union* of f_A and f_B, denoted by $f_A \widetilde{\cup} f_B = f_{A\cup B}$, is defined by $\left(f_A \widetilde{\cup} f_B\right)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

(iii) Let $f_A, f_B \in S(U)$. Then the *soft intersection* of f_A and f_B, denoted by $f_A \widetilde{\cap} f_B = f_{A\cap B}$, is defined by $\left(f_A \widetilde{\cap} f_B\right)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

For $x \in S$, we define

$$A_x = \{(y, z) \in S \times S \mid x \leq y \circ z\}.$$

Definition 9 (see [15]). Let f_A and g_B be two soft sets of an ordered semihypergroup S over U. Then, the uni-soft product, denoted by $f_A \widetilde{*} g_B$, is defined by

$$f_A \widetilde{*} g_B : S \longrightarrow P(U), x \longmapsto \left(f_A \widetilde{*} g_B\right)(x) = \begin{cases} \bigcap\limits_{(y,z)\in A_x} \{f_A(y) \cup g_B(z)\}, & \text{if } A_x \neq \emptyset, \\ U, & \text{if } A_x = \emptyset, \end{cases}$$

for all $x \in S$.

Definition 10 (see [15]). Let $A \subseteq S$. Then the soft characteristic function $\mathcal{S}_A : S \longrightarrow P(U)$ is defined by

$$\mathcal{S}_A(x) := \begin{cases} U & \text{if } x \in A \\ \emptyset & \text{if } x \notin A. \end{cases}$$

The soft set (U_S, S), where $U_S(x) = U$ for all $x \in S$, is called the *identity soft set* over U. For the characteristic soft set $\mathcal{S}_A$ over U, the soft set $\mathcal{S}_A^c$ over U given as follows

$$\mathcal{S}_A^c(x) := \begin{cases} \emptyset & \text{if } x \in A \\ U & \text{if } x \notin A. \end{cases}$$

For an ordered semihypergroup S, the soft set "$\emptyset_S$" of S over U is defined as follows

$$\emptyset_S : S \longrightarrow P(U), x \longmapsto \emptyset_S(x) = \emptyset \text{ for all } x \in S.$$

$\emptyset_S$ is called an *empty soft set* of S over U.

Definition 11 (see [15]). Let f_A be a soft set of an ordered semihypergroup S over U a subset δ such that $\delta \in P(U)$. The δ-exclusive set of f_A is denoted by $e_A(f_A, \delta)$ and defined to be the set

$$e_A(f_A, \delta) = \{x \in S \mid f_A(x) \subseteq \delta\}.$$

2.3. Uni-soft hyperideals of ordered semihypergroups

Definition 12 (see [15]). A soft set $f_A$of an ordered semihypergroup S over U is called *a uni-soft semihypergroup of* S over U if:

$$(\forall x, y \in S) \bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq f_A(x) \cup f_A(y).$$

Definition 13 (see [15]). Let f_A be soft set of an ordered semihypergroup S over U. Then f_A is called a uni-*soft left* (right) *hyperideal* of S over U if it satisfies the following conditions.

(1) $(\forall x, y \in S) \bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq f_A(y) \left(\bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq f_A(x) \right).$

(2) $(\forall x, y \in S)\ x \leq y \Longrightarrow f_A(x) \subseteq f_A(y).$

A soft set f_A of S over U is called a *uni-soft hyperideal* of S over U if it is both a uni-soft left hyperideal and a uni-soft right hyperideal of S over U.

Definition 14 (see [6]). A uni-soft semihypergroup f_A of an ordered semihypergroup S over U is called a uni-soft bi-*hyperideal* of S over U if it satisfies the following conditions.

(1) $(\forall x, y, z \in S) \bigcup_{\alpha \in x \circ y \circ z} f_A(\alpha) \subseteq f_A(x) \cup f_A(z).$

(2) $(\forall x, y \in S) x \leq y \Longrightarrow f_A(x) \subseteq f_A(y).$

3. Uni-soft quasi-hyperideals of ordered semihypergroups

Definition 15. Let f_A be soft set of an ordered semihypergroup S over U. Then f_A is called a uni-soft quasi-*hyperideal* of S over U if it satisfies the following conditions.

(1) $(f_A\widetilde{*}\emptyset_S)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)\,\widetilde{\supseteq}f_A$.

(2) $(\forall x, y \in S)x \leq y \Longrightarrow f_A(x) \subseteq f_A(y)$.

Example 2. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and order relation are defined

$\circ$	a	b	c
a	$\{a\}$	$\{a,b\}$	$\{a,c\}$
b	$\{a\}$	$\{a,b\}$	$\{a,c\}$
c	$\{a\}$	$\{a,b\}$	$\{c\}$

$$\leq := \{(a,a),(b,b),(c,c),(a,b)\}.$$

Suppose $U = \{p,q,r\}$ and $A = \{b,c\}$. Let us define $f_A(a) = \emptyset$, $f_A(b) = \{p,q\}$, and $f_A(c) = \{p,r\}$. Then f_A is a uni-soft quasi-hyperideal of S over U.

Proposition 3. *Let S be an ordered semihypergroup. Then every uni-soft quasi-hyperideal of S over U is a uni-soft semihypergroup of S over U.*

Proof. The proof is straightforward. ■

Theorem 4. *Let f_A be a soft set of an ordered semihypergroup S over U and $\delta \in P(U)$. Then f_A is a uni-soft quasi-hyperideal of S over U if and only if the nonempty δ-exclusive set $e_A(f_A, \delta)$ is a quasi-hyperideal of S.*

Proof. Assume that f_A is a uni-soft quasi-hyperideal of S over U. Let $\delta \in P(U)$ with $\delta \neq \emptyset$. We show that $(e_A(f_A,\delta) \circ S] \cap (S \circ e_A(f_A,\delta)] \subseteq e_A(f_A,\delta)$. Let $a \in (e_A(f_A,\delta) \circ S] \cap (S \circ e_A(f_A,\delta)]$. Then $a \in (e_A(f_A,\delta) \circ S]$ and $a \in (S \circ e_A(f_A,\delta)]$, i.e., $a \leq b \circ s$ and $a \leq k \circ c$ for some $b, c \in e_A(f_A,\delta)$ $s, k \in S$, i.e., $(b,s), (k,c) \in A_a$. This implies that

$$(f_A\widetilde{*}\,\emptyset_S)(a) = \bigcap_{(p,q)\in A_a} \{f_A(p) \cup \emptyset_S(q)\} \subseteq \{f_A(b) \cup \emptyset_S(s)\} = \delta \cup \emptyset = \delta.$$

and

$$(\emptyset_S\widetilde{*}f_A)(a) = \bigcap_{(x,y)\in A_a} \{\emptyset_S(x) \cup f_A(y)\} \subseteq \{\emptyset_S(k) \cup f_A(c)\} = \emptyset \cup \delta = \delta.$$

By assumption, we obtain that

$$f_A(a) \subseteq (f_A\widetilde{*}\emptyset_S)(a)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)(a) \subseteq \delta \cup \delta.$$

Hence $f_A(a) \subseteq \delta$. Thus $a \in e_A(f_A,\delta)$. Therefore $(e_A(f_A,\delta) \circ S] \cap (S \circ e_A(f_A,\delta)] \subseteq e_A(f_A,\delta)$. Let $x \in e_A(f_A,\delta)$ and $y \in S$ with $y \leq x$. So $f_A(y) \subseteq f_A(x) \subseteq \delta$, we obtain $y \in e_A(f_A,\delta)$. Therefore $e_A(f_A,\delta)$ is a quasi-hyperideal of S.

Conversely, we assume that for every $\delta \in P(U)$, $e_A(f_A, \delta)$ is a quasi-hyperideal of S. We show that $(f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A) \widetilde{\supseteq} f_A$. Let $a \in S$. If $A_a = \emptyset$, then it is clear that $(f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A) \widetilde{\supseteq} f_A$. If $A_a \neq \emptyset$, then there exists $x, y \in S$ such that $a \leq x \circ y$. Let $\delta = f_A(x) \cup f_A(y)$. Since $e_A(f_A, \delta)$ is a quasi-hyperideal, $a \leq x \circ y$ and $x, y \in e_A(f_A, \delta)$, we have $a \in (e_A(f_A, \delta) \circ S] \cap (S \circ e_A(f_A, \delta)] \subseteq e_A(f_A, \delta)$. Then $f_A(a) \subseteq \delta$. This means that $f_A(a) \subseteq f_A(x) \cup f_A(y)$ for all $(x, y) \in A_a$. Now we have

$$\begin{aligned}
\left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a) &= (f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) \\
&= \left(\bigcap_{(x,y) \in A_a} \{f_A(x) \cup \emptyset_S(y)\} \right) \\
&\quad \widetilde{\cup} \left(\bigcap_{(x,y) \in A_a} \{\emptyset_S(x) \cup f_A(y)\} \right) \\
&= \left(\bigcap_{(x,y) \in A_a} \{f_A(x) \cup \emptyset\} \right) \widetilde{\cup} \left(\bigcap_{(x,y) \in A_a} (\{f_A(y) \cup \emptyset\}) \right) \\
&= \bigcap_{(x,y) \in A_a} (f_A(x) \cup f_A(y)) \supseteq f_A(a).
\end{aligned}$$

Thus $(f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A) \widetilde{\supseteq} f_A$. Let $a, b \in S$ with $a \leq b$. Since $a \leq b$, $b \in e_A(f_A, \delta)$, let $f_A(b) = \delta$ and $e_A(f_A, \delta)$ is a quasi-hyperideal of S, we get $a \in e_A(f_A, \delta)$ so $f_A(a) \subseteq \delta = f_A(b)$. Hence $f_A(a) \subseteq f_A(b)$. Therefore f_A is a uni-soft quasi-hyperideal of S over U. ■

Example 5. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and order relation are defined

$\circ$	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,b\}$	$\{a\}$	$\{a,d\}$	$\{a\}$
c	$\{a\}$	$\{a,e\}$	$\{a,c\}$	$\{a,c\}$	$\{a,e\}$
d	$\{a\}$	$\{a,b\}$	$\{a,d\}$	$\{a,d\}$	$\{a,b\}$
e	$\{a\}$	$\{a,e\}$	$\{a\}$	$\{a,c\}$	$\{a\}$

$$\leq := \{(a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (a,c), (a,d), (a,e)\}.$$

The covering relation is given below

$$\prec = \{(a,b), (a,c), (a,d), (a,e)\}.$$

It is easy to see that the quasi-hyperideal of S are $\{a\}$, $\{a,b\}$, $\{a,c\}$ $\{a,d\}$, $\{a,e\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{a,b,e\}$, $\{a,c,e\}$ and S.

Suppose $U = \{e_1, e_2, e_3, e_4\}$ and $A = \{b, c, d, e\}$. Let us define $f_A(a) = \emptyset$, $f_A(b) = \{e_1, e_3\}$, $f_A(c) = \{e_1, e_2, e_3, e_4\}$, $f_A(d) = \{e_1, e_3, e_4\}$ and $f_A(e) = \{e_2, e_3, e_4\}$. Then

$$e_A(f_A, \delta) = \begin{cases} \{a\} & \text{if } \delta = \{e_1\} \\ \{a\} & \text{if } \delta = \{e_2\} \\ \{a\} & \text{if } \delta = \{e_3\} \\ \{a\} & \text{if } \delta = \{e_4\} \\ \{a\} & \text{if } \delta = \{e_1, e_2\} \\ \{a, b\} & \text{if } \delta = \{e_1, e_3\} \\ \{a\} & \text{if } \delta = \{e_1, e_4\} \\ \{a\} & \text{if } \delta = \{e_2, e_3\} \\ \{a\} & \text{if } \delta = \{e_2, e_4\} \\ \{a\} & \text{if } \delta = \{e_3, e_4\} \\ \{a, b\} & \text{if } \delta = \{e_1, e_2, e_3\} \\ \{a\} & \text{if } \delta = \{e_1, e_2, e_4\} \\ \{a, b, d\} & \text{if } \delta = \{e_1, e_3, e_4\} \\ \{a, e\} & \text{if } \delta = \{e_2, e_3, e_4\} \\ S & \text{if } \delta = U. \end{cases}$$

So by Theorem 4, f_A is a uni-soft quasi-hyperideal of S over U.

Proposition 6. *Let S be an ordered semihypergroup and Q be the nonempty subset of S. Then Q is a quasi-hyperideal of S if and only if the soft set $\mathcal{S}_Q^c$ is a uni-soft quasi-hyperideal of S over U.*

Proof. Suppose Q is a quasi-hyperideal of an ordered semihypergroup S. Let a be any element of S. If $a \in Q$, then

$$\left(\left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} \mathcal{S}_Q^c\right)\right)(a) \supseteq \emptyset = \mathcal{S}_Q^c(a).$$

If $a \notin Q$, then $\mathcal{S}_Q^c(a) = U$. On the other hand, assume that

$$\left(\left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} \mathcal{S}_Q^c\right)\right)(a) = \emptyset.$$

Then

$$\bigcap_{(x,y)\in A_a} \left\{\mathcal{S}_Q^c(x) \cup \emptyset_S(y)\right\} = \left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right)(a) = \emptyset,$$

and

$$\bigcap_{(x,y)\in A_a} \left\{\emptyset_S(x) \cup \mathcal{S}_Q^c(y)\right\} = \left(\emptyset_S \widetilde{*} \mathcal{S}_Q^c\right)(a) = \emptyset.$$

This implies that there exist elements b, c, d and e of S with $a \leq b \circ c$ and $a \leq d \circ e$ such that $\mathcal{S}_Q^c(b) = \emptyset$ and $\mathcal{S}_Q^c(e) = \emptyset$. Hence $a \leq b \circ c \subseteq Q \circ S \subseteq (Q \circ S]$ and

$a \leq d \circ e \subseteq S \circ Q \subseteq (S \circ Q]$, that is $a \in (Q \circ S] \cap (S \circ Q]$, which is a contradiction that $a \notin Q$. Thus we have

$$\left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} \mathcal{S}_Q^c\right) \widetilde{\supseteq} \mathcal{S}_Q^c.$$

Let $x, y \in S$ with $x \leq y$. If $y \notin Q$ then $\mathcal{S}_Q^c(y) = U$ and so $\mathcal{S}_Q^c(x) \subseteq U = \mathcal{S}_Q^c(y)$. If $y \in Q$ then $\mathcal{S}_Q^c(y) = \emptyset$. Since $x \leq y$ and Q is a quasi-hyperideal of S, we have $x \in Q$ and thus $\mathcal{S}_Q^c(x) = \emptyset = \mathcal{S}_Q^c(y)$. Therefore $\mathcal{S}_Q^c$ is a uni-soft quasi-hyperideal of S over U.

Conversely, let $\mathcal{S}_Q^c$ be a uni-soft quasi-hyperideal of S over U. Let a be any element of $(Q \circ S] \cap (S \circ Q]$. Then there exist elements s and t of S and elements b and c of Q such that $a \leq b \circ s$ and $a \leq t \circ c$. Thus we have

$$\left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right)(a) = \bigcap_{(x,y) \in A_a} \left\{\mathcal{S}_Q^c(x) \cup \emptyset_S(y)\right\} \subseteq \left\{\mathcal{S}_Q^c(b) \cup \emptyset_S(s)\right\} = \emptyset \cup \emptyset = \emptyset,$$

and so

$$\left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right)(a) = \emptyset.$$

Similarly, we have

$$\left(\emptyset_S \widetilde{*} \mathcal{S}_Q^c\right)(a) = \emptyset.$$

Hence

$$\mathcal{S}_Q^c(a) \widetilde{\subseteq} \left(\left(\mathcal{S}_Q^c \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} \mathcal{S}_Q^c\right)\right)(a) = \emptyset \cup \emptyset = \emptyset.$$

Thus $a \in Q$ and so $(Q \circ S] \cap (S \circ Q] \subseteq Q$. Let $x \in S$ and $y \in Q$ be such that $x \leq y$. Then $\mathcal{S}_Q^c(x) \subseteq \mathcal{S}_Q^c(y) = \emptyset$, and thus $x \in Q$. Thus Q is a quasi-hyperideal of S. ∎

Theorem 7. *Let S be an ordered semihypergroup. Then every uni-soft right (resp., left) hyperideal of S over U is a uni-soft quasi-hyperideal of S over U.*

Proof. Let f_A be a uni-soft right hyperideal of S over U. Let $a \in S$, we have

$$\left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a) = (f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a).$$

If $A_a = \emptyset$, it is clear that $(f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) \supseteq f_A(a)$. Let $A_a \neq \emptyset$. Let $(x, y) \in A_a$. We have $a \leq x \circ y$. This mean that $a \leq z$ for some $z \in x \circ y$. Since f_A is a uni-soft right hyperideal of S over U, $f_A(a) \subseteq f_A(z) \subseteq \bigcup_{z \in x \circ y} f_A(z) \subseteq f_A(x)$. It follows that

$$\begin{aligned} f_A(a) &\subseteq \bigcap_{a \leq x \circ y} f_A(x) = \bigcap_{(x,y) \in A_a} \{f_A(x) \cup \emptyset_S(y)\} \\ &= (f_A \widetilde{*} \emptyset_S)(a) \subseteq (f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) = \left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a). \end{aligned}$$

Therefore f_A is a uni-soft quasi-hyperideal of S over U. ∎

The converse of the above Theorem is not true in general. We can illustrate it by the following example.

Example 8. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a\}$
d	$\{a\}$	$\{a,d\}$	$\{a\}$	$\{a\}$

$$\leq:= \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d)\}.$$

Let $U = \{h_1, h_2, h_3\}$ and $A = \{c, d\}$. Define $f_A(a) = \emptyset$, $f_A(b) = \emptyset$, $f_A(c) = \{h_1, h_2\}$ and $f_A(d) = \{h_2\}$. Then f_A is a uni-soft quasi-hyperideal of S over U. But f_A is neither uni-soft right hyperideal nor uni-soft left hyperideal of S over U. Because $\bigcup_{\beta \in b\circ c} f_A(\beta) = f_A(a) \cup f_A(c) = \{h_1, h_2\} \nsubseteq \emptyset = f_A(b)$ and $\bigcup_{\delta \in d\circ b} f_A(\delta) = f_A(a) \cup f_A(d) = \{h_2\} \nsubseteq \emptyset = f_A(b)$.

Theorem 9. *Let S be an ordered semihypergroup. Then every uni-soft quasi-hyperideal of S over U is a uni-soft bi-hyperideal of S over U.*

Proof. Let f_A be a uni-soft quasi-hyperideal of S over U and $x, y \in S$. We show that $\bigcup_{a \in x\circ y\circ z} f_A(a) \subseteq f_A(x) \cup f_A(z)$. Since $a \in x\circ y\circ z \leq x\circ(y\circ z) \Longrightarrow a \leq x\circ w$ for some $w \in y\circ z$, we get

$$(f_A \widetilde{*} \emptyset_S)(a) = \bigcap_{(u,v)\in A_a} \{f_A(u) \cup \emptyset_S(v)\} \subseteq f_A(x) \cup \emptyset_S(w) = f_A(x) \cup \emptyset = f_A(x).$$

Since $a \in x\circ y\circ z \leq (x\circ y)\circ z \Longrightarrow a \leq t\circ z$ for some $t \in x\circ y$. We obtain

$$(\emptyset_S \widetilde{*} f_A)(a) = \bigcap_{(u,v)\in A_a} \{\emptyset_S(u) \cup f_A(v)\} \subseteq \emptyset_S(t) \cup f_A(z) = \emptyset \cup f_A(z) = f_A(z).$$

By assumption we have

$$f_A(a) \subseteq ((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A))(a) = (f_A \widetilde{*} \emptyset_S)(a) \widetilde{(\cup)} (\emptyset_S \widetilde{*} f_A)(a).$$

Hence

$$\bigcup_{a\in x\circ y\circ z} f_A(a) \subseteq (f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) \subseteq f_A(x) \cup f_A(z).$$

Thus $\bigcup_{a\in x\circ y\circ z} f_A(a) \subseteq f_A(x) \cup f_A(z)$. Therefore f_A is a uni-soft bi-hyperideal of S over U. ■

Theorem 10. *In a regular ordered semihypergroup S, the uni-soft quasi-hyperideals of S over U and the uni-soft bi-hyperideals of S over U coincide.*

Proof. Let f_A be a uni-soft bi-hyperideal of S over U. We show that f_A is a uni-soft quasi-hyperideal of S over U. Let $a \in S$. If $A_a = \emptyset$, then it is clear that $\left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a) \supseteq f_A(a)$. If $A_a \neq \emptyset$, then

$$(f_A \widetilde{*} \emptyset_S)(a) = \bigcap_{(x,y)\in A_a} \{f_A(x) \cup \emptyset_S(y)\}$$

and

$$(\emptyset_S \widetilde{*} f_A)(a) = \bigcup_{(u,v)\in A_a} \{\emptyset_S(u) \cup f_A(v)\}.$$

If $(f_A \widetilde{*} \emptyset_S)(a) \supseteq f_A(a)$, then $f_A(a) \subseteq \left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a)$. If $(f_A \widetilde{*} \emptyset_S)(a) \subset f_A(a)$, then there exists $(x,y) \in A_a$ such that $f_A(x) \cup \emptyset_S(y) = f_A(x) \subset f_A(a)$. We claim that $(\emptyset_S \widetilde{*} f_A)(a) \supseteq f_A(a)$. Let $(u,v) \in A_a$. Since S is regular so there exists $w \in S$ such that $a \leq a \circ w \circ a$. It turns out $a \leq x \circ y \circ w \circ u \circ v$, i.e., there exists $b \in x \circ y \circ w \circ u \circ v$ such that $a \leq b$. Since f_A is a uni-soft bi-hyperideal of S over U,

$$f_A(a) \subseteq f_A(b) \subseteq \bigcup_{b\in x\circ\gamma\circ v \text{ and } \gamma\in y\circ w\circ u} f_A(b) \subseteq f_A(x) \cup f_A(v).$$

If $f_A(x) \cup f_A(v) = f_A(x)$, then $f_A(a) \subseteq f_A(x)$. This gives a contradiction. Then $f_A(x) \cup f_A(v) = f_A(v)$ and so $f_A(a) \subseteq f_A(v) = \emptyset_S(u) \cup f_A(v)$ for all $(u,v) \in A_a$. Hence $f_A(a) \subseteq \bigcap_{(u,v)\in A_a} \{\emptyset_S(u) \cup f_A(v)\} = (\emptyset_S \widetilde{*} f_A)(a)$. Now the claim is proved. Therefore $(f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) \supseteq f_A(a)$. Thus $(f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A) \widetilde{\supseteq} f_A$. ■

Lemma 11. *Let S be an ordered semihypergroup and f_A be a soft set of S over U, such that $f_A(a) \subseteq f_A(b)$ for every $a, b \in S$ with $a \leq b$. Then*

(1) *$f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)$ is a uni-soft left hyperideal of S over U,*

(2) *$f_A \widetilde{\cap} (f_A \widetilde{*} \emptyset_S)$ is a uni-soft right hyperideal of S over U.*

Proof. (1) Let $a, b \in S$ and $c \in a \circ b$. We have

$$\begin{aligned}
\left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(c) &= f_A(c) \cap (\emptyset_S \widetilde{*} f_A)(c) \subseteq (\emptyset_S \widetilde{*} f_A)(c) \\
&= \bigcap_{(x,y)\in A_c} \{\emptyset_S(x) \cup f_A(y)\} \\
&= \bigcap_{(x,y)\in A_c} \{\emptyset \cup f_A(y)\} = \bigcap_{(x,y)\in A_c} \{f_A(y)\} \\
&\subseteq f_A(b) \quad (\text{since } c \in a \circ b \text{ and then } (a,b) \in A_c).
\end{aligned}$$

Next we show that $(\emptyset_S \widetilde{*} f_A)(c) \subseteq (\emptyset_S \widetilde{*} f_A)(b)$. Let $A_b \neq \emptyset$ and $(r,s) \in A_b$. Since $(a,b) \in A_c$, we have $(r,s) \in A_b \Longrightarrow b \leq r \circ s \Longrightarrow a \circ b \leq (a \circ r) \circ s \Longrightarrow c \leq t \circ s$ for some $t \in a \circ r$. We have

$$\begin{aligned}(\emptyset_S \widetilde{*} f_A)(c) &= \bigcap_{(p,q) \in A_c} \{\emptyset_S(p) \cup f_A(q)\} \subseteq \emptyset_S(t) \cup f_A(s) \\ &= \emptyset \cup f_A(s) = f_A(s) = \emptyset_S(r) \cup f_A(s).\end{aligned}$$

Thus

$$(\emptyset_S \widetilde{*} f_A)(c) \subseteq \bigcap_{(r,s) \in A_b} \{\emptyset_S(r) \cup f_A(s)\} = (\emptyset_S \widetilde{*} f_A)(b),$$

and then

$$\left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(c) \subseteq \left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)(b)\right).$$

Thus

$$\bigcup_{c \in a \circ b} \left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(c) \subseteq \left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(b).$$

Next we show that for any $a, b \in S$ with $a \leq b$ implies $\left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(a) \subseteq \left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(b)$. Since $A_a \supseteq A_b$, we have $(\emptyset_S \widetilde{*} f_A)(a) \subseteq (\emptyset_S \widetilde{*} f_A)(b)$. Then $f_A(a) \cap (\emptyset_S \widetilde{*} f_A)(a) \subseteq f_A(b) \cap (\emptyset_S \widetilde{*} f_A)(b)$. Thus $\left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right)(a) \subseteq \big(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\big)(b)$. Thus f_A is a uni-soft left hyperideal of S over U.

(2) Similarly we can prove that $f_A \widetilde{\cap} (f_A \widetilde{*} \emptyset_S)$ is a uni-soft right hyperideal of S over U. ■

Lemma 12. *Let S be an ordered semihypergroup and f_A be a soft set of S over U. Then f_A is a uni-soft quasi-hyperideal of S over U if and only if there exist a uni-soft right hyperideal g_B and a uni-soft left hyperideal h_C of S over U such that*

$$f_A = \left(g_B \widetilde{\cup} h_C\right).$$

Proof. By Lemma 11, $f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)$ is a uni-soft left hyperideal of S over U and $f_A \widetilde{\cap} (f_A \widetilde{*} \emptyset_S)$ is a uni-soft right hyperideal of S over U. Moreover, we have,

$$f_A = \left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right) \widetilde{\cup} \left(f_A \widetilde{\cap} (f_A \widetilde{*} \emptyset_S)\right).$$

In fact, by simple calculation we have,

$$\begin{aligned}\left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right) \widetilde{\cup} \left(f_A \widetilde{\cap} (f_A \widetilde{*} \emptyset_S)\right) &= \left(f_A \widetilde{\cap} \left((\emptyset_S \widetilde{*} f_A) \widetilde{\cup} f_A\right)\right) \widetilde{\cap} \left(f_A \widetilde{\cup} (f_A \widetilde{*} \emptyset_S)\right) \\ &\quad \widetilde{\cap} \left((\emptyset_S \widetilde{*} f_A) \widetilde{\cup} (f_A \widetilde{*} \emptyset_S)\right).\end{aligned}$$

Since f_A is a uni-soft quasi-hyperideal of S over U. We have $(f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A) \widetilde{\supseteq} f_A$. Besides $(\emptyset_S \widetilde{*} f_A) \widetilde{\cup} f_A \widetilde{\supseteq} f_A$ and $f_A \widetilde{\cup} (f_A \widetilde{*} \emptyset_S) \widetilde{\supseteq} f_A$. Hence

$$\left(f_A \widetilde{\cap} (\emptyset_S \widetilde{*} f_A)\right) \widetilde{\cup} \left(f_A \widetilde{\cap} (f_A \widetilde{*} \emptyset_S)\right) = f_A.$$

Conversely, let g_B is a uni-soft right hyperideal of S over U and h_C is a uni-soft left hyperideal of S over U such that $f_A = \left(g_B \widetilde{\cup} h_C\right)$. We show that f_A is a uni-soft quasi-hyperideal of S over U. Let $a \in S$, then

$$\left(\left(f_A \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} f_A\right)\right)(a) \widetilde{\supseteq} f_A(a).$$

In fact: Since $\left(\left(f_A \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} f_A\right)\right)(a) = \left(f_A \widetilde{*} \emptyset_S\right)(a) \widetilde{\cup} \left(\emptyset_S \widetilde{*} f_A\right)(a)$. If $A_a = \emptyset$, then $\left(f_A \widetilde{*} \emptyset_S\right)(a) = U = \left(\emptyset_S \widetilde{*} f_A\right)(a)$. So $\left(\left(f_A \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} f_A\right)\right)(a) \widetilde{\supseteq} f_A(a)$. If $A_a \neq \emptyset$, and $(x, y) \in A_a$then $a \leq x \circ y$. Then there exists $b \in x \circ y$ such that $a \leq b$. Since g_B is a uni-soft right hyperideal of S over U and $f_A = \left(g_B \widetilde{\cup} h_C\right)$, we have $g_B(a) \subseteq g_B(b) \subseteq \bigcup_{b \in x \circ y} g_B(b) \subseteq g_B(x) \subseteq f_A(x)$. Now we have $g_B(a) \subseteq f_A(x)$ for all $(x, y) \in A_a$. Hence

$$\begin{aligned} \left(f_A \widetilde{*} \emptyset_S\right)(a) &= \bigcap_{(x,y) \in A_a} \left\{f_A(x) \cup \emptyset_S(y)\right\} = \bigcap_{(x,y) \in A_a} \left\{f_A(x) \cup \emptyset\right\} \\ &= \bigcap_{(x,y) \in A_a} \left\{f_A(x)\right\} \supseteq g_B(a). \end{aligned}$$

Similarly we can show that $\left(\emptyset_S \widetilde{*} f_A\right)(a) \supseteq h_C(a)$. Thus

$$\begin{aligned} \left(\left(f_A \widetilde{*} \emptyset_S\right) \widetilde{\cup} \left(\emptyset_S \widetilde{*} f_A\right)\right)(a) &= \left(f_A \widetilde{*} \emptyset_S\right)(a) \widetilde{\cup} \left(\emptyset_S \widetilde{*} f_A\right)(a) \\ &\supseteq g_B(a) \cup h_C(a) \\ &= \left(g_B \widetilde{\cup} h_C\right)(a) = f_A(a). \end{aligned}$$

Therefore f_A is a uni-soft quasi-hyperideal of S over U. ∎

4. Characterizations of left, right and completely regular ordered semihypergroups in terms of uni-soft quasi-hyperideals

In this section, we introduce the notion of semiprime uni-soft quasi-hyperideals of ordered semihypergroups. We characterize ordered semihypergroups in terms of uni-soft quasi-hyperideals.

An ordered semihypergroup S is called left (resp., right) simple if for every left (resp., right) hyperideal A of S, we have $A = S$ (see [1]).

Definition 16 (see [1]). Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then S is called regular if $a \in (a \circ S \circ a]$.

Lemma 13 (see [1]). *An ordered semihypergroup S is left (resp., right) simple if and only if $(S \circ a] = S$ (resp., $(a \circ S] = S$) for every $a \in S$.*

Example 14 (see [22]). Let $(S,\circ,\leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by

$\circ$	α	β	γ
α	$\{\alpha\}$	$\{\alpha,\beta\}$	$\{\alpha,\beta,\gamma\}$
β	$\{\alpha,\beta\}$	$\{\alpha,\beta,\gamma\}$	$\{\alpha,\beta,\gamma\}$
γ	$\{\alpha,\beta,\gamma\}$	$\{\alpha,\beta,\gamma\}$	$\{\alpha,\beta,\gamma\}$

$$\leq:=\{(\alpha,\alpha),(\beta,\beta),(\gamma,\gamma),(\alpha,\beta),(\alpha,\gamma),(\beta,\gamma)\}.$$

It is easy to see that $(S,\circ,\leq)$ is a left and right simple and also complete regular ordered semihypergroup.

Theorem 15. *An ordered semihypergroup $(S,\circ,\leq)$ is regular, left and right simple if and only if every uni-soft quasi- hyperideal f_A of S over U is a constant function.*

Proof. Let S be regular, left and right simple ordered semihypergroup. Let f_A be a uni-soft quasi- hyperideal of S over U. Let $a\in S$, we consider the set

$$E_\Omega=\left\{e\in S\mid e^2\geq e\right\}.$$

Then $E_\Omega\neq\emptyset$. In fact, since S is regular and $a\in S$, so there exists $x\in S$ such that $a\leq a\circ x\circ a$. So $(a\circ x)^2=(a\circ x)\circ(a\circ x)=(a\circ x\circ a)\circ x\geq a\circ x$, and so $a\circ x\in E_\Omega$ and hence $E_\Omega\neq\emptyset$.

(1) Let $t\in E_\Omega$ then $f_A(e)=f_A(t)$ for every $e\in E_\Omega$. Indeed: Since S is left and right simple, we have $(S\circ t]=S$ and $(t\circ S]=S$. Since $e\in S$, therefore $e\in(S\circ t]$ and $e\in(t\circ S]$ so there exist $x,y\in S$ such that $e\leq x\circ t$ and $e\leq t\circ y$. Hence

$$e^2\leq(x\circ t)\circ(x\circ t)=(x\circ t\circ x)\circ t$$

So there exist $u\in e\circ e$, and $v\in x\circ t\circ x$ such that $u\leq v\circ t$. and we have

$$(v,t)\in A_u,\ \text{and if } e\leq t\circ y$$

then

$$e^2\leq(t\circ y)\circ(t\circ y)=t\circ(y\circ t\circ y),$$

so there exists $w\in y\circ t\circ y$ such that $u\leq t\circ w$ and hence $(t,w)\in A_u$. Since $A_u\neq\emptyset$ and f_A is a uni-soft quasi-hyperideal of S over U, we have

$$\begin{aligned}
f_A(u) &\subseteq \left((f_A\widetilde{*}\emptyset_S)\widetilde{\cup}(\emptyset_S\widetilde{*}f_A)\right)(u)=(f_A\widetilde{*}\emptyset_S)(u)\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)(u)\\
&=\left[\bigcap_{(y_1,z_1)\in A_u}\{f_A(y_1)\cup\emptyset_S(z_1)\}\right]\cup\left[\bigcap_{(y_2,z_2)\in A_u}\{\emptyset_S(y_2)\cup f_A(z_2)\}\right]\\
&\subseteq\{f_A(t)\cup\emptyset_S(w)\}\cup\{\emptyset_S(v)\cup f_A(t)\}\\
&=\{f_A(t)\cup\emptyset\}\cup\{\emptyset\cup f_A(t)\}=f_A(t)\cup f_A(t)=f_A(t).
\end{aligned}$$

Since $e \in E_\Omega$, we have $u \geq e$ and f_A is a uni-soft quasi-hyperideal of S over U, we have $f_A(e) \subseteq f_A(u)$. Thus $f_A(e) \subseteq f_A(t)$. Since S is left and right simple and $e \in S$, we have $(S \circ e] = S$ and $(e \circ S] = S$. Since $t \in S$, we have $t \leq z \circ e$ and $t \leq e \circ s$ for some $z, s \in S$. If $t \leq z \circ e$, then

$$t^2 \leq (z \circ e) \circ (z \circ e) = (z \circ e \circ z) \circ e,$$

then there exist $\alpha \in t \circ t$ and $\beta \in z \circ e \circ z$ such that $\alpha \leq \beta \circ e$. So $(\beta, e) \in A_\alpha$. If $t \leq e \circ s$ then

$$t^2 \leq (e \circ s) \circ (e \circ s) = e \circ (s \circ e \circ s).$$

Then there exists $\gamma \in s \circ e \circ s$ such that $\alpha \leq e \circ \gamma$. So $(e, \gamma) \in A_\alpha$. Since $A_\alpha \neq \emptyset$, we have,

$$\begin{aligned} f_A(\alpha) &\subseteq \left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(\alpha) = (f_A \widetilde{*} \emptyset_S)(\alpha) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(\alpha) \\ &= \left[\bigcap_{(p_1,q_1) \in A_\alpha} \{f_A(p_1) \cup \emptyset_S(q_1)\}\right] \cup \left[\bigcap_{(p_2,q_2) \in A_\alpha} \{\emptyset_S(p_2) \cup f_A(q_2)\}\right] \\ &\subseteq \{f_A(e) \cup \emptyset_S(\gamma)\} \cup \{\emptyset_S(\beta) \cup f_A(e)\} \\ &= \{f_A(e) \cup \emptyset\} \cup \{\emptyset \cup f_A(e)\} = f_A(e) \cup f_A(e) = f_A(e). \end{aligned}$$

Since $t \in E_\Omega$ we have $\alpha \geq t$ and since f_A is a uni-soft quasi-hyperideal of S over U, we have $f_A(t) \subseteq f_A(\alpha)$. Thus $f_A(t) \subseteq f_A(e)$ and therefore $f_A(t) = f_A(e)$ for every $e \in E_\Omega$.

(2) Let $a \in S$ then $f_A(t) = f_A(a)$ for every $t \in E_\Omega$. Since $a \in S$ and S is regular, there exists $x \in S$ such that $a \leq a \circ x \circ a$. Then it follows,

$$(a \circ x)^2 = (a \circ x) \circ (a \circ x) = (a \circ x \circ a) \circ x \geq a \circ x$$

and

$$(x \circ a)^2 = (x \circ a) \circ (x \circ a) = x \circ (a \circ x \circ a) \geq x \circ a.$$

Then $a \circ x$, $x \circ a \in E_\Omega$. Thus there exist $h_1 \in a \circ x$, and $h_2 \in x \circ a$. Hence $h_1, h_2 \in E_\Omega$. Then by (1) we have $f_A(h_1) = f_A(t)$ and $f_A(h_2) = f_A(t)$. Since $(a \circ x) \circ (a \circ x \circ a) \geq a \circ x \circ a \geq a$, and $(a \circ x \circ a) \circ (x \circ a) \geq a \circ x \circ a \geq a$ then there exists $h_3 \in a \circ x \circ a$ such that $(h_1, h_3) \in A_a$ and $(h_3, h_2) \in A_a$. Since $A_a \neq \emptyset$ and f_A is a uni-soft quasi-hyperideal of S over U, we have

$$\begin{aligned}
f_A(a) &\subseteq \left((f_A\widetilde{*}\emptyset_S)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)\right)(a) = (f_A\widetilde{*}\emptyset_S)(a)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)(a)\\
&= \left[\bigcap_{(y_1,z_1)\in A_a}\{f_A(y_1)\cup\emptyset_S(z_1)\}\right]\cup\left[\bigcap_{(y_2,z_2)\in A_u}\{\emptyset_S(y_2)\cup f_A(z_2)\}\right]\\
&\subseteq \{f_A(h_1)\cup\emptyset_S(h_3)\}\cup\{\emptyset_S(h_3)\cup f_A(h_2)\}\\
&= \{f_A(h_1)\cup\emptyset\}\cup\{\emptyset\cup f_A(h_2)\}\\
&= f_A(h_1)\cup f_A(h_2) = f_A(t)\cup f_A(t) = f_A(t)\,.
\end{aligned}$$

Since S is left and right simple we have $(S\circ a]=S$, $(a\circ S]=S$, Since $t\in S$ we have $t\in(S\circ a]$ and $t\in(a\circ S]$. Then $t\le p\circ a$ and $t\le a\circ q$ for some $p,q\in S$. Then $(p,a)\in A_t$ and $(a,q)\in A_t$. Since $A_t\neq\emptyset$ and f_A is a uni-soft quasi-hyperideal of S over U, we have

$$\begin{aligned}
f_A(t) &\subseteq \left((f_A\widetilde{*}\emptyset_S)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)\right)(t) = (f_A\widetilde{*}\emptyset_S)(t)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)(t)\\
&= \left[\bigcap_{(y_1,z_1)\in A_t}\{f_A(y_1)\cup\emptyset_S(z_1)\}\right]\cup\left[\bigcap_{(y_2,z_2)\in A_t}\{\emptyset_S(y_2)\cup f_A(z_2)\}\right]\\
&\subseteq \{f_A(a)\cup\emptyset_S(q)\}\cup\{\emptyset_S(p)\cup f_A(a)\}\\
&= \{f_A(a)\cup\emptyset\}\cup\{\emptyset\cup f_A(a)\} = f_A(a)\cup f_A(a) = f_A(a)\,.
\end{aligned}$$

Since S is left and right simple we have $(S\circ a]=S$, $(a\circ S]=S$, Since $t\in S$ we have $t\in(S\circ a]$ and $t\in(a\circ S]$. Then $t\le p\circ a$ and $t\le a\circ q$ for some $p,q\in S$. Then $(p,a)\in A_t$ and $(a,q)\in A_t$. Since $A_t\neq\emptyset$, we have

$$\begin{aligned}
f_A(t) &\subseteq \left((f_A\widetilde{*}\emptyset_S)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)\right)(t) = (f_A\widetilde{*}\emptyset_S)(t)\,\widetilde{\cup}\,(\emptyset_S\widetilde{*}f_A)(t)\\
&= \left[\bigcap_{(y_1,z_1)\in A_t}\{f_A(y_1)\cup\emptyset_S(z_1)\}\right]\cup\left[\bigcap_{(y_2,z_2)\in A_t}\{\emptyset_S(y_2)\cup f_A(z_2)\}\right]\\
&\subseteq \{f_A(a)\cup\emptyset_S(q)\}\cup\{\emptyset_S(p)\cup f_A(a)\}\\
&= \{f_A(a)\cup\emptyset\}\cup\{\emptyset\cup f_A(a)\} = f_A(a)\cup f_A(a) = f_A(a)\,.
\end{aligned}$$

Conversely, let $a\in S$ then the set $(a\circ S]=S$ is a quasi-hyperideal of S. In fact, $((a\circ S]\circ S]\cap(S\circ(S\circ a]]\subseteq(a\circ S]\cap(S\circ a\circ S]\subseteq(a\circ S]$. If $x\in(a\circ S]$ and $S\ni y\le x\in(a\circ S]$, then $y\in((a\circ S]]=(a\circ S]$. Since $(a\circ S]$ is a quasi-hyperideal of S. Then by Proposition 6, $\mathcal{S}^c_{(a\circ S]}$ of $(a\circ S]$ defined by

$$\mathcal{S}^c_{(a\circ S]}: S\longrightarrow P(U)\,,\ x\longrightarrow\begin{cases}\emptyset, & \text{if } x\in(a\circ S]\\ U, & \text{if } x\notin(a\circ S]\end{cases}$$

is a uni-soft quasi-hyperideal of S over U. By hypothesis $\mathcal{S}^c_{(a\circ S]}$ is a constant function, that is there exists $\delta \subseteq U$ such that

$$\mathcal{S}^c_{(a\circ S]}(x) = \delta \text{ for every } x \in S.$$

Let $(a \circ S] \subseteq S$ and a be an element of S such that $a \notin (a \circ S]$, then $\mathcal{S}^c_{(a\circ S]}(a) = U$. On the other hand, since $a^2 \subseteq (a \circ S]$. Then we have $\mathcal{S}^c_{(a\circ S]}(a^2) = \emptyset$. A contradiction to the fact that $\mathcal{S}^c_{(a\circ S]}$ is a constant function. Thus $(a \circ S] = S$. By symmetry we can prove that $(S \circ a] = S$. Since $a \in S$ and $(a \circ S] = S = (S \circ a]$, we have $a \in (a \circ S] = (a \circ (a \circ S]] \subseteq (a \circ S \circ a]$. Thus S is regular. ■

Definition 17 (see [22])**.** An ordered semihypergroup S is called completely regular if it is regular, left regular and right regular.

Lemma 16 (see [22])**.** *Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then the following statements are equivalent:*

(1) *S is completely regular.*

(2) *$A \subseteq \left(A^2 \circ S \circ A^2\right]$ for all $A \subseteq S$.*

(3) *$a \in \left(a^2 \circ S \circ a^2\right]$ for all $a \in S$.*

Theorem 17. *An ordered semihypergroup $(S, \circ, \leq)$ is completely regular if and only if for every uni-soft quasi-hyperideal f_A of S over U, we have*

$$f_A(a) = \bigcup_{\alpha \in a\circ a} f_A(\alpha) \ \textit{for all } a \in S.$$

Proof. Let S be a completely regular ordered semihypergroup and f_A is a uni-soft quasi-hyperideal of S over U and let $a \in S$. Since S is left and right regular, we have $a \in \left(S \circ a^2\right]$ and $a \in \left(a^2 \circ S\right]$. Then there exists $x, y \in S$ such that $a \leq x \circ a^2$ and $a \leq a^2 \circ y$. So there exists $\alpha \in a^2$ such that $a \leq x \circ \alpha$ and $a \leq \alpha \circ y$. Then $(x, \alpha) \in A_a$, $(\alpha, y) \in A_a$. Since $A_a \neq \emptyset$, we have

$$\begin{aligned}
f_A(a) &\subseteq \left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a) = (f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) \\
&= \left[\bigcap_{(y_1,z_1)\in A_a} \{f_A(y_1) \cup \emptyset_S(z_1)\}\right] \cup \left[\bigcap_{(y_2,z_2)\in A_a} \{\emptyset_S(y_2) \cup f_A(z_2)\}\right] \\
&\subseteq \{f_A(\alpha) \cup \emptyset_S(y)\} \cup \{\emptyset_S(x) \cup f_A(\alpha)\} \\
&= \{f_A(\alpha) \cup \emptyset\} \cup \{\emptyset \cup f_A(\alpha)\} = f_A(\alpha) \cup f_A(\alpha) = f_A(\alpha). \\
f_A(a) &\subseteq f_A(\alpha).
\end{aligned}$$

Therefore $f_A(a) \subseteq \bigcup_{\alpha\in a\circ a} f_A(\alpha)$. Since every uni-soft quasi-hyperideal is a uni-soft semihypergroup. Therefore f_A is a uni-soft semihypergroup of S over U. Hence $\bigcup_{\alpha\in a\circ a} f_A(\alpha) \subseteq f_A(a) \cup f_A(a) = f_A(a)$. Thus $\bigcup_{\alpha\in a\circ a} f_A(\alpha) = f_A(a)$.

Conversely, let $a \in S$. We consider the quasi-hyperideal $Q\left(a^2\right)$ of S generated by a^2. That is $Q\left(a^2\right) = \left(a^2 \cup \left(\left(a^2 \circ S\right] \cap \left(S \circ a^2\right]\right)\right]$. By Proposition 6, $\mathcal{S}^c_{Q(a^2)}$ is a uni-soft quasi-hyperideal of S over U. By hypothesis we have $\mathcal{S}^c_{Q(a^2)}(a) = \bigcup_{\alpha\in a\circ a} \mathcal{S}^c_{Q(a^2)}(\alpha)$. Since $a^2 \subseteq Q\left(a^2\right)$ and $Q\left(a^2\right) = \left(a^2 \cup \left(\left(a^2 \circ S\right] \cap \left(S \circ a^2\right]\right)\right]$, we have $\bigcup_{\alpha\in a\circ a} \mathcal{S}^c_{Q(a^2)}(\alpha) = \emptyset$, then $\mathcal{S}^c_{Q(a^2)}(a) = \emptyset$ and we have $a \in Q\left(a^2\right) = \left(a^2 \cup \left(\left(a^2 \circ S\right] \cap \left(S \circ a^2\right]\right)\right]$. Then $a \leq a^2$ or $a \leq a^2 \circ x$ and $a \leq y \circ a^2$ for some $x, y \in S$. If $a \leq a^2$ then $a \leq a^2 = a \circ a \leq a^2 \circ a^2 = a \circ a \circ a^2 \leq a^2 \circ a \circ a^2$ and so $a \in \left(a^2 \circ S \circ a^2\right]$. If $a \leq a^2 \circ x$ and $a \leq y \circ a^2$ then $a \leq \left(a^2 \circ x\right) \circ \left(y \circ a^2\right) = a^2 \circ (x \circ y) \circ a^2$. Since $x \circ y \subseteq S$. Hence $a \in \left(a^2 \circ S \circ a^2\right]$. Thus by Lemma 16, S is completely regular. ■

Definition 18. Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A is a uni-soft quasi-hyperideal of S over U. Then f_A is called semiprime uni-soft quasi-hyperideal of S over U if

$$f_A(a) \subseteq \bigcup_{\alpha\in a\circ a} f_A(\alpha) \text{ for all } a \in S.$$

Theorem 18. *An ordered semihypergroup $(S, \circ, \leq)$ is completely regular if and only if for every uni-soft quasi-hyperideal f_A of S over U is semiprime.*

Proof. Let S be completely regular and f_A is a uni-soft quasi-hyperideal of S over U. Let $a \in S$. Then $f_A(a) \subseteq \bigcup_{\alpha\in a\circ a} f_A(\alpha)$ for all $a \in S$. In fact, since S is left and right regular, and $a \in S$, then there exists $x, y \in S$ such that $a \leq x \circ a^2$ and $a \leq a^2 \circ y$. So there exists $\alpha \in a^2$ such that $a \leq x \circ \alpha$ and $a \leq \alpha \circ y$. Then $(x, \alpha) \in A_a$, $(\alpha, y) \in A_a$. Since $A_a \neq \emptyset$, then

$$\begin{aligned}
f_A(a) &\subseteq \left((f_A \widetilde{*} \emptyset_S) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)\right)(a) = (f_A \widetilde{*} \emptyset_S)(a) \widetilde{\cup} (\emptyset_S \widetilde{*} f_A)(a) \\
&= \left[\bigcap_{(p_1,q_1)\in A_a} \{f_A(p_1) \cup \emptyset_S(q_1)\}\right] \cup \left[\bigcap_{(p_2,q_2)\in A_a} \{\emptyset_S(p_2) \cup f_A(q_2)\}\right] \\
&\subseteq \{f_A(\alpha) \cup \emptyset_S(y)\} \cup \{\emptyset_S(x) \cup f_A(\alpha)\} \\
&= \{f_A(\alpha) \cup \emptyset\} \cup \{\emptyset \cup f_A(\alpha)\} = f_A(\alpha) \cup f_A(\alpha) = f_A(\alpha) \\
&\subseteq f_A(\alpha)
\end{aligned}$$

Therefore $f_A(a) \subseteq \bigcup_{\alpha\in a\circ a} f_A(\alpha)$.

Conversely, let f_A be a uni-soft quasi-hyperideal of S over U such that $f_A(a) \subseteq \bigcup_{\alpha\in a\circ a} f_A(\alpha)$ for all $a \in S$. We consider the quasi-hyperideal $Q\left(a^2\right)$ of S generated by a^2. That is $Q\left(a^2\right) = \left(a^2 \cup \left(\left(a^2 \circ S\right] \cap \left(S \circ a^2\right]\right)\right]$. By Proposition 6, $\mathcal{S}^c_{Q(a^2)}$ is a uni-soft quasi-hyperideal of S over U. By hypothesis we have $\mathcal{S}^c_{Q(a^2)}(a) \subseteq \bigcup_{\alpha\in a\circ a} \mathcal{S}^c_{Q(a^2)}(\alpha)$. Since $a^2 \subseteq Q\left(a^2\right)$ and $Q\left(a^2\right) = \left(a^2 \cup \left(\left(a^2 \circ S\right] \cap\right.\right.$

$\left(S \circ a^2\right]\right)\big]$, we have $\bigcup_{\alpha \in a^2} \mathcal{S}^c_{Q(a^2)}(\alpha) = \emptyset$, and $\mathcal{S}^c_{Q(a^2)}(a) = \emptyset$ and we have $a \in Q\left(a^2\right) = \left(a^2 \cup \left(\left(a^2 \circ S\right] \cap \left(S \circ a^2\right]\right)\right]$. Then $a \leq a^2$ or $a \leq a^2 \circ x$ and $a \leq y \circ a^2$ for some x, $y \in S$. If $a \leq a^2$ then $a \leq a^2 = a \circ a \leq a^2 \circ a^2 = a \circ a \circ a^2 \leq a^2 \circ a \circ a^2$ and so $a \in \left(a^2 \circ S \circ a^2\right]$. If $a \leq a^2 \circ x$ and $a \leq y \circ a^2$ then $a \leq \left(a^2 \circ x\right) \circ \left(y \circ a^2\right) = a^2 \circ (x \circ y) \circ a^2$. Since $x \circ y \subseteq S$. Hence $a \in \left(a^2 \circ S \circ a^2\right]$. Thus by Lemma 16, S is completely regular. ■

5. Conclusion

The theme of this paper is to enhance the understanding of ordered semihypergroups in context of uni-soft quasi-hyperideals. In the present paper we introduced the notion of uni-soft quasi-hyperideals. Some related properties are investigated. Moreover we proved that every uni-soft left (right) hyperideal is a uni-soft quasi-hyperideal. Furthermore we discussed the characterizations of left (right) simple and completely regular ordered semihypergroups in terms of uni-soft quasi-hyperideals. Finally we defined semiprime uni-soft quasi-hyperideals and characterized completely regular ordered semihypergroup in terms of uni-soft quasi-hyperideals. We hope that this work would helps for further study of hyperstructures and their applications.

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Received 3 March 2020
Revised 13 September 2020
Accepted 16 September 2020