

COMPLETELY PRIME HYPERIDEALS OF TERNARY SEMIHYPERGROUPS

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Abstract

In this article, we introduce the notions of pseudosymmetric hyperideals and globally idempotent ternary semihypergroups and present various examples for them. We prove that if a ternary semihypergroup is globally idempotent, then every maximal hyperideal is a prime hyperideal. Also we study some properties of prime, completely prime and pseudosymmetric hyperideals of a ternary semihypergroup and characterize them. The interrelation among them is considered in ternary semihypergroups.

Keywords: pseudosymmetric, globally idempotent, leftsimple ternary semihypergroup.

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1. INTRODUCTION

A key role is being played by algebraic structures with so many applications in mathematics. The various disciplines are computer sciences, information sciences, theoretical physics, coding theory etc. The algebraic concept of semigroups was widely investigated by Clifford and Preston [3]. Operations of ternary algebra are considered by mathematicians in 19th century like Cayley [4]. He introduced

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the cubic matrix which later was generalized by Karpranov in 1994 [14]. n -ary structures which are generalizations of ternary structures are raising hopes with their applications in coding theory, physics etc. The concept of n -ary group was developed by Dörnte [10] in 1928. Many authors [7] investigated sets with n -ary operation with different properties. These have various applications in various branches of science. Many applications of ternary structures are described in physics by Kerner [15]. Hypercubes induced n -ary structures have their application in cryptology, error coding and detecting coding theory.

Lehmer [16] introduced ternary algebraic system. He also investigated algebraic systems known as triplexes which later turnout as commutative ternary-groups. Ideal theory was developed by Sioson [22] in ternary semigroups in 1965. He developed some notions of regular ternary semigroup. He also characterized them with the notion of quasi ideals. In 1995, Dixit and Dewan [9] introduced and studied some properties of ideals and quasi-(bi-) ideals in ternary semigroups and in [23, 25], some other results on ternary semigroups are provided. The ideals of n -ary semigroups are studied by Dudek [11, 12].

Hyperstructure theory was introduced in 1934 by Marty [17]. He introduced the concept of hypergroup. After that several mathematicians studied this subject. A variety of applications are the results of hyperstructure theory. The main concepts of the theory of hyperstructures was found in [5, 6, 8, 13, 18, 19, 26]. We know that in a classical algebraic structure the composition of two elements is an element. In algebraic hyperstructures, the composition two elements is a set.

Basar [2] developed some results on gamma semihypergroups. In [20, 21] Nichefranca *et al.* studied the pseudosymmetric ideals in semigroups. In [24], Sarala studied the pseudosymmetric ideals in ternary semigroups.

In this paper we introduce pseudosymmetric hyperideals and globally idempotent in ternary semihypergroups. An effort was made to study the notions of completely prime, prime and pseudosymmetric hyperideals by providing various examples for them and the interrelation among them was also established. We also prove that if a ternary semihypergroup is globally idempotent, then every maximal hyperideal is a prime hyperideal. Pseudosymmetric ideals were first developed by Anjaneyulu [1] in semigroups. Based on this concept we found some results in ternary semihypergroups. In future, We may continue our study by applying these results on semihypergroups, gamma semihypergroups etc.

2. PRELIMINARIES

We first recall the fundamental terms as well as definitions from the ternary semihypergroup concept.

Definition 2.1 [19]. Let $T \neq \phi$ and $P^*(T)$ be the family of all nonempty subsets of T . A hyperoperation on T is a map $\circ : T \times T \rightarrow P^*(G)$ and the pair (T, \circ) is known as a hypergroupoid.

If J and K are nonempty subsets of T , then we denote $J \circ K = \bigcup_{j \in J, k \in K} j \circ k$; $x \circ j = \{x\} \circ j$ and $j \circ x = j \circ \{x\}$. A hypergroupoid (T, \circ) is known as a semihypergroup if $\forall d, e, f \in T$, we have $(d \circ e) \circ f = d \circ (e \circ f)$ which means $\bigcup_{r \in d \circ e} r \circ f = \bigcup_{s \in e \circ f} d \circ s$.

Definition 2.2 [18]. A map $g : T \times T \times T \rightarrow P^*(T)$ is called a ternary hyperoperation on the set T , where $T \neq \emptyset$ and $P^*(T) = P(T) \setminus \{\phi\}$ shows the family of all nonempty subsets of (T, g) .

Definition 2.3 [8]. A ternary hypergroupoid is called a pair (T, g) where g is a ternary hyperoperation on the set T . If Q, R, S are nonempty subsets of T , then we define $g(Q, R, S) = \bigcup_{q \in Q, r \in R, s \in S} g(q, r, s)$.

Definition 2.4 [13]. A ternary hypergroupoid (T, g) is called a ternary semihypergroup if for all $y_1, y_2, y_3, y_4, y_5 \in T$, we have

$$g(g(y_1, y_2, y_3), y_4, y_5) = g(y_1, g(y_2, y_3, y_4), y_5) = g(y_1, y_2, g(y_3, y_4, y_5)).$$

Definition 2.5 [8]. Let (T, g) be a ternary semihypergroup. Then T is called ternary hypergroup if $\forall p, q, r \in T, \exists a, b, c \in T \ni r \in g(a, p, q) \cap g(p, b, q) \cap g(p, q, c)$.

Definition 2.6 [18]. Let us suppose that (T, g) is a ternary semihypergroup and $\emptyset \neq H \subseteq T$. Then H is called a ternary subsemihypergroup of $T \Leftrightarrow g(H, H, H) \subseteq H$.

Different examples of ternary semihypergroup can be found in [7, 8, 18, 19].

Definition 2.7 [13]. Suppose (T, g) be a ternary semihypergroup and $\phi \neq A \subseteq T$.

- (i) If $g(T, T, A) \subseteq A$, then A is called as left hyperideal of T .
- (ii) If $g(A, T, T) \subseteq A$, then A is called as a right hyperideal of T .
- (iii) If $g(T, A, T) \subseteq A$, then A is called as a lateral hyperideal of T .
- (iv) If it is lateral, right and left hyperideal of T , then it is a hyperideal of T .
- (v) If it is right and left hyperideal of T , then it is two sided hyperideal of T .

3. SOME CHARACTERIZATIONS OF COMPLETELY PRIME HYPERIDEALS

In this section, we present some results on completely prime hyperideals. We also introduce the notions of abelian, globally idempotent and pseudosymmetric ternary semihypergroups and their properties are investigated.

Definition 3.1. A left hyperideal B of a ternary semihypergroup (T, g) is called a proper left hyperideal of T if $B \neq T$.

Definition 3.2 [18]. A left hyperideal B of a ternary semihypergroup (T, g) is called maximal left hyperideal if B is a proper left hyperideal of T and is not well contained in each proper left hyperideal of T .

Definition 3.3. Let (T, g) be a ternary semihypergroup and $\phi \neq B \subseteq T$. The lowest left hyperideal of T containing B is called as a left ternary hyperideal of T generated by B .

Theorem 3.4. *The left hyperideal of a ternary semihypergroup (T, g) generated by a nonempty subset B is the intersection of all left hyperideals of T which contains B .*

Proof. Assume that Δ is the family of all left hyperideals of T which contains B . As T alone acts as a left hyperideal of T which contains B , $T \in \Delta$. So $\Delta \neq \phi$.

Suppose that $\{Q_\alpha\}_{\alpha \in \Delta}$ is a set of left hyperideals of T and assume $Q = \bigcap_{\alpha \in \Delta} Q_\alpha$. Put $p \in Q$; $q, r \in T$. Here $p \in Q \Rightarrow p \in \bigcap_{\alpha \in \Delta} Q_\alpha \Rightarrow p \in Q_\alpha$ for each $\alpha \in \Delta$. $p \in Q_\alpha$; $q, r \in T$; Q_α is a left hyperideal of $T \Rightarrow g(q, r, p) \subseteq Q_\alpha$. $g(q, r, p) \subseteq Q_\alpha \forall \alpha \in \Delta \Rightarrow g(q, r, p) \subseteq \bigcap_{\alpha \in \Delta} Q_\alpha \Rightarrow g(q, r, p) \subseteq Q$.

Therefore Q acts as a left hyperideal of T . Put K be a left hyperideal of T containing B . Obviously $B \subseteq K$. Hence $K \in \Delta \Rightarrow Q \subseteq K$. Therefore Q is the left hyperideal of T generated by B . ■

Definition 3.5. A left hyperideal A of a ternary semihypergroup (T, g) is called the principal left hyperideal generated by p if A is a left hyperideal generated by $\{p\}$ for some $p \in T$. It is symbolized by $\langle p \rangle_l$.

Theorem 3.6. *If (T, g) is a ternary semihypergroup and $a \in T$, then $\langle a \rangle_l = \{a\} \cup g(T, T, a)$.*

Proof. Let $y, z \in T$; $x \in \{a\} \cup g(T, T, a)$. $x \in \{a\} \cup g(T, T, a) \Rightarrow x = a$ (or) $x \in g(u, v, a)$ for some $u, v \in T$. If $x = a$, then $g(y, z, x) = g(y, z, a) \subseteq g(T, T, a) \subseteq \{a\} \cup g(T, T, a)$. If $x \in g(u, v, a)$, then $g(y, z, x) = g(y, z, g(u, v, a)) = g(g(y, z, u), v, a) \subseteq g(T, T, a) \subseteq \{a\} \cup g(T, T, a)$.

Therefore $g(y, z, x) \subseteq \{a\} \cup g(T, T, a)$ and hence $\{a\} \cup g(T, T, a)$ is a left hyperideal of T . Let L be a left hyperideal of T containing a .

Let $x \in \{a\} \cup g(T, T, a)$. Then $x = \{a\}$ or $x \in g(u, v, a)$ for some $u, v \in T$. If $x = \{a\}$, then $x = a \in L$. If $x \in g(u, v, a)$, then $x \in g(u, v, a) \subseteq L$. Therefore $\{a\} \cup g(T, T, a) \subseteq L$. Hence $\{a\} \cup g(T, T, a)$ is the smallest left hyperideal which contains ' a '. Therefore $\langle a \rangle_l = \{a\} \cup g(T, T, a)$. ■

Now we recall the definition of left simple ternary semihypergroup and characterize it.

Definition 3.7 [18]. A ternary semihypergroup (T, g) is called a left simple ternary semihypergroup if T is its only left hyperideal.

Theorem 3.8. A ternary semihypergroup (T, g) is a left simple ternary semihypergroup iff $g(T, T, a) = T \forall a \in T$.

Proof. Assume that (T, g) is a left simple ternary semihypergroup and $a \in T$. Let $s \in g(T, T, a)$ $s \in g(T, T, a) \Rightarrow s \in g(v, w, a); v, w \in T$. Now $g(u, t, s) \subseteq g(u, t, g(v, w, a)) = g(g(u, t, v), w, a) \subseteq g(T, T, a) \Rightarrow g(T, T, a)$ is a left hyperideal of T . As T is a left simple ternary semihypergroup, $g(T, T, a) = T \forall a \in T$.

Conversely assume that $g(T, T, a) = T \forall a \in T$. Let I be a left hyperideal of T . Let $i \in I$, then $i \in T$. By supposition $g(T, T, i) = T$. Let $t \in T$. Then $t \in g(T, T, i) \Rightarrow t \in g(u, v, i)$ for some $u, v \in T$. $i \in I; u, v \in T$ and I is a left hyperideal of $T \Rightarrow g(u, v, i) \subseteq I \Rightarrow t \in I$. Therefore $T \subseteq I$. Clearly $I \subseteq T$ and hence $I = T$. Hence T is the only left hyperideal of T . Therefore T is left simple ternary semihypergroup. ■

Here we introduce abelian ternary semihypergroup and provide some examples.

Definition 3.9. A ternary semihypergroup (T, g) is called abelian if $g(d, e, f) = g(e, f, d) = g(f, d, e) = g(e, d, f) = g(f, e, d) = g(d, f, e) \forall d, e, f \in T$.

Example 3.10. Let $T = \{-i, 0, i\}$ and $g(x, y, z) = (x \star y) \star z \forall x, y, z \in T$ where \star is defined by the usual multiplication. Then (T, g) is a abelian ternary semihypergroup.

Now we recall the definitions of completely prime and completely semiprime hyperideals and characterize them.

Definition 3.11 [13]. A hyperideal I of a ternary semihypergroup (T, g) is called completely prime hyperideal of T if $l, m, n \in T$ and $g(l, m, n) \subseteq I$ implies either $l \in I$ or $m \in I$ or $n \in I$.

A hyperideal I of a ternary semihypergroup (T, g) is called completely semiprime of T if $g(x, x, x) \subseteq I$ implies $x \in I$ for any element $x \in I$.

Example 3.12. Let (H, \circ) be a ternary semihypergroup on $H = \{p, q, r, s, t\}$ with the hyper operation ' \circ ' given the following table:

\circ	p	q	r	s	t
p	$\{p, q\}$	$\{p, q\}$	r	r	r
q	$\{p, q\}$	$\{p, q\}$	r	r	r
r	$\{p, q\}$	$\{p, q\}$	r	r	r
s	$\{p, q\}$	$\{p, q\}$	r	$\{s, t\}$	s
t	$\{p, q\}$	$\{p, q\}$	r	s	t

Clearly $I = \{r, s, t\}$ is completely prime hyperideal of H .

Example 3.13. Let $H = \{0, p, q, r\}$ and $f(a, b, c) = (a \star b) \star c$ for all $a, b, c \in H$ where \star is defined by the table:

\star	0	p	q	r
0	0	0	0	0
p	0	0	p	0
q	0	0	q	0
r	0	p	0	r

Then $\{H, f\}$ is a ternary semihypergroup. Clearly $A_1 = \{0, p, q\}$, $A_2 = \{0, p, r\}$, $A_3 = H$, are all completely prime hyperideals.

Theorem 3.14. *An ideal A of a ternary semihypergroup (T, g) is completely prime hyperideal $\Leftrightarrow g(b_1, b_2, \dots, b_n) \subseteq T$; n is an odd natural number, $n \geq 3$ $g(b_1, b_2, \dots, b_n) \subseteq A \Rightarrow b_i \in A$ for some $i = 1, 2, \dots, n$.*

Proof. Suppose that A is a completely prime hyperideal of T . Let $g(b_1, b_2, \dots, b_n) \subseteq T$ where n is an odd natural number, $n \geq 3$ and $g(b_1, b_2, \dots, b_n) \subseteq A$.

If $n = 3$, then $g(b_1, b_2, b_3) \subseteq A \Rightarrow b_1 \in A$ or $b_2 \in A$ or $b_3 \in A$. If $n = 5$, then $g(b_1, b_2, b_3, b_4, b_5) \subseteq A \Rightarrow g(b_1, b_2, b_3) \subseteq A$ or $b_4 \in A$ or $b_5 \in A \Rightarrow b_1 \in A$ or $b_2 \in A$ or $b_3 \in A$ or $b_4 \in A$ or $b_5 \in A$.

Therefore the result is true for $n \geq 3$ (n is an odd natural number), then $g(b_1, b_2, \dots, b_n) \subseteq A \Rightarrow b_i \in A$ for some $i = 1, 2, \dots, n$. The converse part is trivial. ■

Theorem 3.15. *A hyperideal I of a ternary semihypergroup (T, g) is a completely prime hyperideal iff $T \setminus I$ is either ternary subsemihypergroup of T or $T \setminus I = \phi$.*

Proof. Let us assume that I is a completely prime hyperideal of T and $T \setminus I \neq \phi$. Put $l, m, n \in T \setminus I$. Then $l \notin I, m \notin I, n \notin I$. Suppose if possible $g(l, m, n) \not\subseteq T \setminus I$. Then $g(l, m, n) \subseteq I$. Since I is completely prime hyperideal, either $l \in I$ or $m \in I$ or $n \in I$. It is a contradiction. Therefore $g(l, m, n) \subseteq T \setminus I$. Hence $T \setminus I$ is a ternary subsemihypergroup of T . Conversely consider $T \setminus I$ is a ternary subsemihypergroup of T or $T \setminus I = \phi$. If $T \setminus I = \phi$, then $T = I$ and therefore I is completely prime hyperideal. Assume that $T \setminus I$ is a ternary subsemihypergroup of T . Put $l, m, n \in T$ and $g(l, m, n) \subseteq I$.

Suppose if possible $l \notin I, m \notin I$ and $n \notin I$. Then $l \in T \setminus I, m \in T \setminus I, n \in T \setminus I$. Since $T \setminus I$ is a ternary subsemihypergroup, $g(l, m, n) \subseteq T \setminus I$ and hence $g(l, m, n) \not\subseteq I$. It contradicts our assumption. Hence either $l \in I$ or $m \in I$ or $n \in I$. Therefore I is a completely prime hyperideal of T . ■

Now we recall the definition of prime hyperideal and characterize it.

Definition 3.16 [18]. A hyperideal I of a ternary semihypergroup T , is known as a prime hyperideal of T if Q, R, S are hyperideals of T and $g(Q, R, S) \subseteq I \Rightarrow Q \subseteq I$ or $R \subseteq I$ or $S \subseteq I$.

Example 3.17. Let $H = \{p, q, r, s, t, u\}$ and $f(a, b, c) = a \star b \star c$ for all $a, b, c \in H$; where \star is defined by the table:

\star	p	q	r	s	t	u
p	p	$\{p, q\}$	r	$\{r, s\}$	t	$\{t, u\}$
q	q	q	s	s	u	u
r	r	$\{r, s\}$	r	$\{r, s\}$	r	$\{r, s\}$
s	s	s	s	s	s	s
t	t	$\{t, u\}$	r	$\{r, s\}$	t	$\{t, u\}$
u	u	u	s	s	u	u

Then $\{H, f\}$ is a ternary semihypergroup. Clearly $I_1 = \{r, s\}$, $I_2 = \{r, s, t, u\}$ and H are prime.

Theorem 3.18. *A hyperideal A of a ternary semihypergroup (T, g) is prime iff $g(Q_1, Q_2, \dots, Q_n) \subseteq T$, n is an odd natural number, $n \geq 3$, $g(Q_1, Q_2, \dots, Q_n) \subseteq A \Rightarrow Q_i \in A$ for some $i = 1, 2, \dots, n$.*

Proof. Straight forward. ■

Now we define globally idempotent ternary semihypergroup and characterize it.

Definition 3.19. A hyperideal A of a ternary semihypergroup (T, g) is called globally idempotent if $g(A^n) = g(A) \forall$ odd natural numbers $n, n \geq 3$. A ternary semihypergroup (T, g) is called globally idempotent ternary semihypergroup if every hyperideal of T is globally idempotent.

Theorem 3.20. *If (T, g) is a globally idempotent ternary semihypergroup, then every maximal hyperideal of T is prime hyperideal of T .*

Proof. Let M be a maximal hyperideal of T . Put P, Q, R be three hyperideals of T such that $g(P, Q, R) \subseteq M$. Suppose $P \not\subseteq M, Q \not\subseteq M, R \not\subseteq M$. $P \not\subseteq M \Rightarrow M \cup P$ is a hyperideal of T and $M \subset M \cup P \subseteq T$. Since M is maximal hyperideal, $M \cup P = T$. Similarly $Q \not\subseteq M \Rightarrow M \cup Q = T$ and $R \not\subseteq M \Rightarrow M \cup R = T$. Now $T = g(T, T, T) = g(M \cup P, M \cup Q, M \cup R) \subseteq M \Rightarrow T \subseteq M$. Then $M = T$. It is a contradiction. Therefore either $P \subseteq M$ or $Q \subseteq M$ or $R \subseteq M$. Hence M is a prime hyperideal. ■

Now we introduce pseudosymmetric hyperideals and characterize them.

Definition 3.21. A hyperideal I of a ternary semieypergroup (T, g) is known as a pseudosymmetric hyperideal, if $d, e, f \in T$; $g(d, e, f) \subseteq I \Rightarrow g(d, s, e, t, f) \subseteq I \forall s, t \in T$.

Example 3.22. Let $T = \{0, d, e, f\}$ and $g(x, y, z) = x \star y \star z$ for all $x, y, z \in T$; where \star is defined by the table:

\star	0	d	e	f
0	0	0	0	0
d	0	0	d	0
e	0	0	e	0
f	0	d	d	f

Then $\{T, g\}$ is a ternary semihypergroup. Clearly $I_1 = \{0\}$, $I_2 = \{0, d\}$, $I_3 = \{0, d, f\}$, $I_4 = \{0, d, e\}$ and $I_5 = T$ are all pseudosymmetric hyperideals.

Definition 3.23. A ternary semihypergroup (T, g) is said to be pseudosymmetric provided every hyperideal is pseudosymmetric.

Example 3.24. Put $T = \{e, f, g\}$ and $g(x, y, z) = (x \star y) \star z \forall x, y, z \in T$; where \star is defined by the table:

\star	e	f	g
e	$\{e, g\}$	$\{f, g\}$	g
f	g	g	g
g	g	g	g

Then $\{T, g\}$ is a ternary semihypergroup. Hyperideals of T are $A_1 = \{g\}$, $A_2 = \{f, g\}$, $A_3 = T$. These hyperideals are pseudosymmetric. Hence (T, g) is a pseudosymmetric ternary semihypergroup.

Theorem 3.25. Every prime hyperideal in a pseudosymmetric ternary semihypergroup (T, g) is completely prime.

Proof. Let P be any prime hyperideal in a pseudosymmetric ternary semihypergroup (T, g) and let $g(l, m, n) \subseteq P$ for some $l, m, n \in T$. Since T is pseudosymmetric ternary semihypergroup, P is a pseudosymmetric hyperideal. Therefore $g(l, s, m, t, n) \subseteq P \forall s, t \in T$. Hence $g(\langle l \rangle, \langle m \rangle, \langle n \rangle) \subseteq P$. So either $\langle l \rangle \subseteq P$ or $\langle m \rangle \subseteq P$ or $\langle n \rangle \subseteq P$. Then either $l \in P$ or $m \in P$ or $n \in P$. So P is a completely prime hyperideal. ■

Theorem 3.26. Let (T, g) be a ternary semihypergroup and A be a prime hyperideal of T , then A is pseudosymmetric hyperideal iff A is a completely prime.

Proof. Put A be a pseudosymmetric hyperideal of (T, g) . If $g(l, m, n) \subseteq A$ for some $l, m, n \in T$. Then $g(l, s, m, t, n) \subseteq A \forall s, t \in T$. Since A is prime we have $l \in A$ or $m \in A$ or $n \in A$. This shows that A is completely prime. The converse part is easy to observe. We hence omit the proof. ■

Remark 3.27. If A is a hyperideal in a pseudosymmetric ternary semihypergroup (T, g) , then $\sqrt{A} = \{y \in T/g(y^n) \subseteq A \text{ for some odd positive integer } n, n \geq 3\}$ where \sqrt{A} is the intersection of all prime hyperideals containing A .

Theorem 3.28. *In a pseudosymmetric ternary semihypergroup (T, g) , a maximal hyperideal M is prime iff $M = \sqrt{M}$.*

Proof. If M is maximal hyperideal, then M of T is a prime hyperideal, then by Theorem 3.25, M is completely prime. Let M be a completely prime hyperideal of a ternary semihypergroup T . Assume that $x \in T$ and $g(x^3) \subseteq M$. Since M is completely prime hyperideal of T , $x \in M$. Therefore M is a completely semiprime hyperideal. So $M = \sqrt{M}$. Conversely if $M = \sqrt{M}$, then by Remark 3.27 M is a prime hyperideal of T . ■

4. CONCLUSION

In this paper we introduce pseudosymmetric hyperideals and globally idempotent in ternary semihypergroups. An effort was made to study the notions of completely prime, prime and pseudosymmetric hyperideals by providing various examples for them. The interrelation between completely prime, prime and pseudosymmetric hyperideals in ternary semihypergroup was established. We also found that if a ternary semihypergroup is globally idempotent, then every maximal hyperideal is a prime hyperideal. In future, We may continue our study by applying these results on semihypergroups, gamma hypergroups etc.

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