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NEAR RINGS CHARACTERIZED BY INTUITIONISTIC FUZZY BI IDEALS

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Abstract

Generalizing the concept of fuzzy bi ideals of near rings, the notion of intuitionistic fuzzy bi ideals of near rings is introduced. We also characterize and investigate some related properties of intuitionistic fuzzy bi ideals of near rings.

Keywords: near rings, fuzzy bi ideals, intuitionistic fuzzy subnear ring, intuitionistic fuzzy ideals, intuitionistic fuzzy bi ideals.

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1. Introduction

After introducing the notion of fuzzy set by Zadeh [16] in 1965, the fuzzy set theory has been used for many applications in the field of mathematics and somewhere else. Rosenfeld [13] introduced fuzzy subgroups and gave some of its properties. Liu [9] has studied fuzzy ideals of a ring and many researchers [6, 7, 8, 11] are engaged in extending the concept. The notion of near ring was introduced by Pilz [12] in 1977 and that of quasi-ideal in near ring was introduced by Yakabe

[15]. In 1991, Abou-Zaid [1] introduced the idea of fuzzy subnear rings and ideals in near rings.

Furthermore, Chelvam and Ganeasan [3] introduced the concept of bi ideals in near rings. Manikantan [10] introduced the idea of fuzzy bi ideals of near rings and discussed some of its properties. The idea of intuitionistic fuzzy set (IFS) was initiated by Atanassov [2] as a generalization of the idea of fuzzy set. Young Bae Jun [5] proposed the idea of intuitionistic fuzzy bi-ideals of ordered semigroups. In 2004, Zhan Jianming and Ma Xueling [4] introduced the notion of intuitionistic fuzzy ideals of near rings. The notion of intuitionistic fuzzy ideal of near rings was introduced by Sharma [14]. In this paper, we study the intuitionistic fuzzification of the notion of bi ideals in near rings. We give some characterizations of intuitionistic fuzzy bi ideals in near rings.

2. PRELIMINARIES

Definition 2.1. A non-empty set $\mathbb{N}$ with two binary operations "+" and "." is called a near ring if it satisfies the following axioms:

(i) $(\mathbb{N}, +)$ is a group.
(ii) $(\mathbb{N}, .)$ is a semigroup.
(iii) $x.(y+z) = x.y + x.z$ for all $x, y, z \in \mathbb{N}$.

We will use the word "near ring" to mean "left near ring". We denote xy instead of $x.y$.

Definition 2.2. An ideal of a near ring $\mathbb{N}$ is a subset $\mathcal{L}$ of $\mathbb{N}$ such that

1. $(\mathcal{L}, +)$ is normal subgroup of $(\mathbb{N},+)$.
2. $\mathbb{N}\mathcal{L} \subseteq \mathcal{L}$.
3. $(x+a)y - xy \in \mathcal{L}$ for all $x, y \in \mathbb{N}$ and $a \in \mathcal{L}$

Note that $\mathcal{L}$ is a left ideal of $\mathbb{N}$ if $\mathcal{L}$ satisfies (1) and (2), and $\mathcal{L}$ is right ideal of $\mathbb{N}$ if $\mathcal{L}$ satisfies (1) and (3).

Definition 2.3. Let $\mathcal{W}$ be a semigroup of a near ring $\mathbb{N}$. Then $\mathcal{W}$ is called a bi ideal of $\mathbb{N}$ if $\mathcal{W}\mathbb{N}\mathcal{W} \cap (\mathcal{W}\mathbb{N})^*\mathcal{W} \subseteq \mathcal{W}$.

Definition 2.4. Let $\mathcal{X}$ be a non-empty set. A mapping $\psi : \mathcal{X} \to [0,1]$ is a fuzzy set in $\mathcal{X}$. The complement of ψ, denoted by ψ^c, is the fuzzy set in $\mathcal{X}$ given by $\psi^c(x) = 1 - \psi(x)$ for all $x \in \mathcal{X}$.

Definition 2.5. A fuzzy set ψ in $\mathbb{N}$ is a fuzzy subnear ring of $\mathbb{N}$ if for all $x, y \in \mathbb{N}$.

1. $\psi(x-y) \geq \min\{\psi(x), \psi(y)\}$ and

2. $\psi(xy) \geq \min\{\psi(x), \psi(y)\}$.

Definition 2.6. A fuzzy set ψ in $\mathbb{N}$ is a fuzzy bi ideal of $\mathbb{N}$ if for all $x, y \in \mathbb{N}$.

1. $\psi(x - y) \geq \min\{\psi(x), \psi(y)\}$ and
2. $\psi(xyz) \geq \min\{\psi(x), \psi(z)\}$ for all $x, y, z \in \mathbb{N}$.

3. Intuitionistic Fuzzy Bi Ideals of Near Rings

In this section, we introduce the notion of intuitionistic fuzzy bi ideals of near rings and discuss some of its properties.

Definition 3.1. An intuitionistic fuzzy set A in a non-empty set $\mathcal{X}$ is an object having the form $A = \{(x, \psi_A(x), \chi_A(x))|x \in \mathcal{X}\}$, where the functions $\psi_A : \mathcal{X} \to [0, 1]$ and $\chi_A : \mathcal{X} \to [0, 1]$ indicate the degree of membership and the degree of non-membership of each element $x \in \mathcal{X}$ to the set A, respectively, and $0 \leq \psi_A(x) + \chi_A(x) \leq 1$ for all $x \in \mathcal{X}$.

We will use the symbol $A = (\psi_A, \chi_A)$ to mean the intuitionistic fuzzy set $A = \{x, \psi_A(x), \chi_A(x)|x \in \mathcal{X}\}$.

Definition 3.2. An intuitionistic fuzzy set $A = (\psi_A, \chi_A)$in $\mathbb{N}$ is an intuitionistic fuzzy subnear ring of $\mathbb{N}$ if for all $x, y \in \mathbb{N}$.

1. $\psi_A(x - y) \geq \min\{\psi_A(x), \psi_A(y)\}$
2. $\psi_A(xy) \geq \min\{\psi_A(x), \psi_A(y)\}$
3. $\chi_A(x - y) \leq \max\{\chi_A(x), \chi_A(y)\}$
4. $\chi_A(xy) \leq \max\{\chi_A(x), \chi_A(y)\}$.

Definition 3.3. An intuitionistic fuzzy set $A = (\psi_A, \chi_A)$ in $\mathbb{N}$ is an intuitionistic fuzzy bi ideal of $\mathbb{N}$ if for all $x, y, z \in \mathbb{N}$.

1. $\psi_A(x - y) \geq \min\{\psi_A(x), \psi_A(y)\}$
2. $\psi_A(xyz) \geq \min\{\psi_A(x), \psi_A(z)\}$
3. $\chi_A(x - y) \leq \max\{\chi_A(x), \chi_A(y)\}$
4. $\chi_A(xyz) \leq \max\{\chi_A(x), \chi_A(z)\}$.

Definition 3.4. Let $A_i = (\psi_{A_i}, \chi_{A_i})$ be an intuitionistic fuzzy bi ideals of near rings $\mathbb{N}_i$ for $i = 1, 2, 3, \ldots, n$. Then the intuitionistic fuzzy direct product of A_i is a function

$$(\psi_{A_1} \times \psi_{A_2} \times \cdots \times \psi_{A_n}) : \mathbb{N}_1 \times \mathbb{N}_2 \times \cdots \times \mathbb{N}_n \to [0, 1],$$
$$(\chi_{A_1} \times \chi_{A_2} \times \cdots \times \chi_{A_n}) : \mathbb{N}_1 \times \mathbb{N}_2 \times \cdots \times \mathbb{N}_n \to [0, 1],$$

defined by

$$(\psi_{A_1} \times \psi_{A_2} \times \cdots \times \psi_{A_n})(x_1, x_2, \ldots, x_n) = \min\{\psi_{A_1}(x_1), \psi_{A_2}(x_2), \ldots, \psi_{A_n}(x_n)\}$$
$$(\chi_{A_1} \times \chi_{A_2} \times \cdots \times \chi_{A_n})(x_1, x_2, \ldots, x_n) = \max\{\chi_{A_1}(x_1), \chi_{A_2}(x_2), \ldots, \chi_{A_n}(x_n)\}.$$

Theorem 3.1. *Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of regular near ring $\mathbb{N}$. Then, every $A = (\psi_A, \chi_A)$ is an intuitionistic fuzzy subnear ring of $\mathbb{N}$.*

Proof. Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of $\mathbb{N}$ and $s, t \in \mathbb{N}$. Since $\mathbb{N}$ is regular, there exist $x \in \mathbb{N}$ such that s=sxs. Then

$$\begin{aligned}
\psi_A(st) &= \psi_A((sxs)t) \\
&= \psi_A(s(xs)t) \\
&\geq \min\{\psi_A(s), \psi_A(t)\} and \\
\chi_A(st) &= \chi_A((sxs)t) \\
&= \chi_A(s(xs)t) \\
&\leq \max\{\chi_A(s), \chi_A(t)\}.
\end{aligned}$$

Thus $A = (\psi_A, \chi_A)$ is an intuitionistic fuzzy subnear ring of $\mathbb{N}$. ■

Proposition 3.2. *Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal in a strongly regular near ring $\mathbb{N}$. Then $\psi_A(x) = \psi_A(x^2)$ and $\chi_A(x) = \chi_A(x^2)$ for all $x \in \mathbb{N}$.*

Proof. Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of $\mathbb{N}$ and $x \in \mathbb{N}$. Since, $\mathbb{N}$ is a strongly regular, there exist $y \in \mathbb{N}$ such that $x = x^2yx^2$. Then

$$\begin{aligned}
\psi_A(x) &= \psi_A(x^2yx^2) \\
&\geq \min\{\psi_A(x^2), \psi_A(x^2)\} \\
&= \psi_A(x^2) \\
&\geq \min\{\psi_A(x), \psi_A(x)\} \\
&= \psi_A(x) and \\
\chi_A(x) &= \chi_A(x^2yx^2) \\
&\leq \max\{\chi_A(x^2), \chi_A(x^2)\} \\
&= \chi_A(x^2) \\
&\leq \max\{\chi_A(x), \chi_A(x)\} \\
&= \chi_A(x).
\end{aligned}$$

Hence $\psi_A(x) = \psi_A(x^2)$ and $\chi_A(x) = \chi_A(x^2)$. ■

Theorem 3.3. *Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of $\mathbb{N}$, then the set $\mathbb{N}_A = \{x \in \mathbb{N}/A(x) = A(0)\}$ is an intuitionistic fuzzy bi ideal of $\mathbb{N}$.*

Proof. Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of $\mathbb{N}$ and $x, y \in \mathbb{N}$, then $A(x) = A(0)$ and $A(y) = A(0)$. Suppose $x, y, z \in \mathbb{N}_A$. Then

$$\begin{aligned} &\psi_A(x) = \psi_A(y) = \psi_A(z) = \psi_A(0) \quad \text{and} \\ &\chi_A(x) = \chi_A(y) = \chi_A(z) = \chi_A(0). \end{aligned}$$

Since ψ_A is a fuzzy bi ideal of $\mathbb{N}$,

$$\begin{aligned} \psi_A(x-y) &\geq \min\{\psi_A(x), \psi_A(y)\} \\ &\geq \min\{\psi_A(0), \psi_A(0)\} = \psi_A(0). \end{aligned}$$

Since χ_A is a fuzzy bi ideal of $\mathbb{N}$,

$$\begin{aligned} \chi_A(x-y) &\leq \max\{\chi_A(x), \chi_A(y)\} \\ &\leq \max\{\chi_A(0), \chi_A(0)\} = \chi_A(0). \end{aligned}$$

Thus $x - y \in \mathbb{N}_A$.

$$\begin{aligned} \psi_A(xyz) &\geq \min\{\psi_A(x), \psi_A(z)\} \\ &\geq \min\{\psi_A(0), \psi_A(0)\} = \psi_A(0) \quad \text{and} \\ \chi_A(xyz) &\leq \max\{\chi_A(x), \chi_A(z)\} \\ &\leq \max\{\chi_A(0), \chi_A(0)\} = \chi_A(0). \end{aligned}$$

Thus $xyz \in \mathbb{N}_A$. Therefore, $\mathbb{N}_A$ is an intuitionistic fuzzy bi ideal of $\mathbb{N}$. ■

Theorem 3.4. *Let $A_i = (\psi_{A_i}, \chi_{A_i})$ be an intuitionistic fuzzy bi ideals of near rings $\mathbb{N}_i$, then their direct product is also an intuitionistic fuzzy bi ideal of near ring.*

Proof. Let $A_i = (\psi_{A_i}, \chi_{A_i})$ be an intuitionistic fuzzy bi ideals of near rings $\mathbb{N}_i$ for $i = 1, 2, \ldots, n$.

Let $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n)$ and $z = (z_1, z_2, \ldots, z_n) \in \mathbb{N}_1 \times \mathbb{N}_2 \times \cdots \times \mathbb{N}_n$.

$$\begin{aligned} \psi_{A_i}(x-y) &= \psi_{A_i}((x_1, x_2, \ldots, x_n) - (y_1, y_2, \ldots, y_n)) \\ &= \psi_{A_i}(x_1 - y_1, x_2 - y_2, \ldots, x_n - y_n) \\ &= \min\{\psi_{A_1}(x_1 - y_1), \psi_{A_2}(x_2 - y_2), \ldots, \psi_{A_n}(x_n - y_n)\} \end{aligned}$$

$$
\begin{aligned}
&\geq \min\{\min\{\psi_{A_1}(x_1), \psi_{A_1}(y_1)\}, \min\{\psi_{A_2}(x_2), \psi_{A_2}(y_2)\}, \ldots, \\
&\quad \min\{\psi_{A_n}(x_n), \psi_{A_n}(y_n)\}\} \\
&\geq \min\{\min\{\psi_{A_1}(x_1), \psi_{A_2}(x_2), \ldots, \psi_{A_n}(x_n)\}, \min\{\psi_{A_1}(y_1), \psi_{A_2}(y_2), \\
&\quad \ldots, \psi_{A_n}(y_n)\}\} \\
&= \min\{(\psi_{A_1} \times \psi_{A_2} \times \cdots \times \psi_{A_n})(x_1, x_2, \ldots, x_n), \\
&\quad (\psi_{A_1} \times \psi_{A_2} \times \cdots \times \psi_{A_n})(y_1, y_2, \ldots, y_n)\}
\end{aligned}
$$

$$\psi_{A_i}(x-y) \geq \min\{\psi_{A_i}(x), \psi_{A_i}(y)\} and$$

$$
\begin{aligned}
\chi_{A_i}(x-y) &= \chi_{A_i}((x_1, x_2, \ldots, x_n) - (y_1, y_2, \ldots, y_n)) \\
&= \chi_{A_i}(x_1 - y_1, x_2 - y_2, \ldots, x_n - y_n) \\
&= \max\{\chi_{A_1}(x_1 - y_1), \chi_{A_2}(x_2 - y_2), \ldots, \chi_{A_n}(x_n - y_n)\} \\
&\leq \max\{\max\{\chi_{A_1}(x_1), \chi_{A_1}(y_1)\}, \max\{\chi_{A_2}(x_2), \chi_{A_2}(y_2)\}, \ldots, \\
&\quad \max\{\chi_{A_n}(x_n), \chi_{A_n}(y_n)\}\} \\
&\leq \max\{\max\{\chi_{A_1}(x_1), \chi_{A_2}(x_2), \ldots, \chi_{A_n}(x_n)\}, \max\{\chi_{A_1}(y_1), \chi_{A_2}(y_2), \\
&\quad \ldots, \chi_{A_n}(y_n)\}\} \\
&= \max\{(\chi_{A_1} \times \chi_{A_2} \times \cdots \times \chi_{A_n})(x_1, x_2, \ldots, x_n), \\
&\quad (\chi_{A_1} \times \chi_{A_2} \times \cdots \times \chi_{A_n})(y_1, y_2, \ldots, y_n)\} \\
\chi_{A_i}(x-y) &\leq \max\{\chi_{A_i}(x), \chi_{A_i}(y)\}
\end{aligned}
$$

$$
\begin{aligned}
\psi_{A_i}(xyz) &= \psi_{A_i}((x_1, x_2, \ldots, x_n)(y_1, y_2, \ldots, y_n)(z_1, z_2, \ldots, z_n)) \\
&= \psi_{A_i}(x_1 y_1 z_1, x_2 y_2 z_2, \ldots, x_n y_n z_n) \\
&= \min\{\psi_{A_1}(x_1 y_1 z_1), \psi_{A_2}(x_2 y_2 z_2), \ldots, \psi_{A_n}(x_n y_n z_n)\} \\
&\geq \min\{\min\{\psi_{A_1}(x_1), \psi_{A_1}(z_1)\}, \min\{\psi_{A_2}(x_2), \psi_{A_2}(z_2)\}, \ldots, \\
&\quad \min\{\psi_{A_n}(x_n), \psi_{A_n}(z_n)\}\} \\
&\geq \min\{\min\{\psi_{A_1}(x_1), \psi_{A_2}(x_2), \ldots, \psi_{A_n}(x_n)\}, \min\{\psi_{A_1}(z_1), \psi_{A_2}(z_2), \\
&\quad \ldots, \psi_{A_n}(z_n)\}\} \\
&= \min\{(\psi_{A_1} \times \psi_{A_2} \times \cdots \times \psi_{A_n})(x_1, x_2, \ldots, x_n), \\
&\quad (\psi_{A_1} \times \psi_{A_2} \times \cdots \times \psi_{A_n})(z_1, z_2, \ldots, z_n)\} \\
\psi_{A_i}(xyz) &\geq \min\{\psi_{A_i}(x), \psi_{A_i}(z)\} and
\end{aligned}
$$

$$
\begin{aligned}
\chi_{A_i}(xyz) &= \chi_{A_i}((x_1, x_2, \ldots, x_n)(y_1, y_2, \ldots, y_n)(z_1, z_2, \ldots, z_n)) \\
&= \chi_{A_i}(x_1 y_1 z_1, x_2 y_2 z_2, \ldots, x_n y_n z_n) \\
&= \max\{\chi_{A_1}(x_1 y_1 z_1), \chi_{A_2}(x_2 y_2 z_2), \ldots, \chi_{A_n}(x_n y_n z_n)\}
\end{aligned}
$$

$$\begin{aligned}
&\leq \max\{\max\{\chi_{A_1}(x_1), \chi_{A_1}(z_1)\}, \max\{\chi_{A_2}(x_2), \chi_{A_2}(z_2)\}, \ldots, \\
&\quad \max\{\chi_{A_n}(x_n), \chi_{A_n}(z_n)\}\} \\
&\leq \max\{\max\{\chi_{A_1}(x_1), \chi_{A_2}(x_2), \ldots, \chi_{A_n}(x_n)\}, \max\{\chi_{A_1}(z_1), \chi_{A_2}(z_2), \\
&\quad \ldots, \chi_{A_n}(z_n)\}\} \\
&= \max\{(\chi_{A_1} \times \chi_{A_2} \times \cdots \times \chi_{A_n})(x_1, x_2, \ldots, x_n), \\
&\quad (\chi_{A_1} \times \chi_{A_2} \times \cdots \times \chi_{A_n})(z_1, z_2, \ldots, z_n)\} \\
\chi_{A_i}(xyz) &\leq \max\{\chi_{A_i}(x), \chi_{A_i}(z)\}
\end{aligned}$$

Hence, the direct product of intuitionistic fuzzy bi ideals of near rings is also an intuitionistic fuzzy bi ideal of near ring. ∎

Theorem 3.5. *Let $\{A_i\} = \{(\psi_{A_i}, \chi_{A_i})/i \in \Omega\}$ be a family of intuitionistic fuzzy bi ideals of near ring $\mathbb{N}$, then the set $\cap_{i\in\Omega}\psi_{A_i}$ and $\cup_{i\in\Omega}\chi_{A_i}$ are also an intuitionistic fuzzy bi ideal of $\mathbb{N}$,where Ω is any index set.*

Proof. Let $\{A_i\} = \{(\psi_{A_i}, \chi_{A_i})/i \in \Omega\}$ be a family of intuitionistic fuzzy bi ideals of near ring $\mathbb{N}$.

Let $x, y, z \in \mathbb{N}$ and $\psi_A = \cap\psi_{A_i}$, $\chi_A = \cup\chi_{A_i}$ then

$$\psi_A(x) = \cap\psi_{A_i}(x) = (\inf \psi_{A_i})(x) = \inf \psi_{A_i}(x)$$
$$\chi_A(x) = \cup\chi_{A_i}(x) = (\sup \chi_{A_i})(x) = \sup \chi_{A_i}(x)$$

$$\begin{aligned}
\psi_A(x-y) &= \inf \psi_{A_i}(x-y) \\
&\geq \inf\{\min\{\psi_{A_i}(x), \psi_{A_i}(y)\}\} \\
&= \min\{\inf \psi_{A_i}(x), \inf \psi_{A_i}(y)\} \\
&= \min\{\cap\psi_{A_i}(x), \cap\psi_{A_i}(y)\} \\
\psi_A(x-y) &= \min\{\psi_A(x), \psi_A(y)\} and
\end{aligned}$$

$$\begin{aligned}
\chi_A(x-y) &= \sup \chi_{A_i}(x-y) \\
&\leq \sup\{\max\{\chi_{A_i}(x), \chi_{A_i}(y)\}\} \\
&= \max\{\sup \chi_{A_i}(x), \sup \chi_{A_i}(y)\} \\
&= \max\{\cup\chi_{A_i}(x), \cup\chi_{A_i}(y)\} \\
\chi_A(x-y) &= \max\{\chi_A(x), \chi_A(y)\}
\end{aligned}$$

$$\begin{aligned}
\psi_A(xyz) &= \inf \psi_{A_i}(xyz) \\
&\geq \inf\{\min\{\psi_{A_i}(x), \psi_{A_i}(z)\}\} \\
&= \min\{\inf \psi_{A_i}(x), \inf \psi_{A_i}(z)\}
\end{aligned}$$

$$\begin{aligned} &= \min\{\cap\psi_{A_i}(x), \cap\psi_{A_i}(z)\} \\ \psi_A(xyz) &= \min\{\psi_A(x), \psi_A(z)\} \quad \text{and} \end{aligned}$$

$$\begin{aligned} \chi_A(xyz) &= \sup \chi_{A_i}(xyz) \\ &\leq \sup\{\max\{\chi_{A_i}(x), \chi_{A_i}(z)\}\} \\ &= \max\{\sup \chi_{A_i}(x), \sup \chi_{A_i}(z)\} \\ &= \max\{\cup\chi_{A_i}(x), \cup\chi_{A_i}(z)\} \\ \chi_A(xyz) &= \max\{\chi_A(x), \chi_A(z)\} \end{aligned}$$

Hence the set $\cap_{i\in\Omega}\psi_{A_i}$ and $\cup_{i\in\Omega}\chi_{A_i}$ are an intuitionistic fuzzy bi ideal of $\mathbb{N}$. ∎

Theorem 3.6. Let $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of $\mathbb{N}$ then $A^c = (\psi_A^c, \chi_A^c)$ is also an intuitionistic fuzzy bi ideal of $\mathbb{N}$.

Proof. Let $x, y, z \in \mathbb{N}$ and $A = (\psi_A, \chi_A)$ be an intuitionistic fuzzy bi ideal of $\mathbb{N}$, then

$$\begin{aligned} \psi_A^c(x-y) &= 1 - \psi_A(x-y) \\ &\leq 1 - \min\{\psi_A(x), \psi_A(y)\} \\ &= \max\{1 - \psi_A(x), 1 - \psi_A(y)\} \\ \psi_A^c(x-y) &= \max\{\psi_A^c(x), \chi_A^c(y)\} and \end{aligned}$$

$$\begin{aligned} \chi_A^c(x-y) &= 1 - \chi_A(x-y) \\ &\geq 1 - \max\{\chi_A(x), \chi_A(y)\} \\ &= \min\{1 - \chi_A(x), 1 - \chi_A(y)\} \\ \chi_A^c(x-y) &= \min\{\chi_A^c(x), \chi_A^c(y)\} \end{aligned}$$

$$\begin{aligned} \psi_A^c(xyz) &= 1 - \psi_A(xyz) \\ &\leq 1 - \min\{\psi_A(x), \psi_A(z)\} \\ &= \max\{1 - \psi_A(x), 1 - \psi_A(z)\} \\ \psi_A^c(xyz) &= \max\{\psi_A^c(x), \chi_A^c(z)\} and \end{aligned}$$

$$\begin{aligned} \chi_A^c(xyz) &= 1 - \chi_A(xyz) \\ &\geq 1 - \max\{\chi_A(x), \chi_A(z)\} \\ &= \min\{1 - \chi_A(x), 1 - \chi_A(z)\} \\ \chi_A^c(xyz) &= \min\{\chi_A^c(x), \chi_A^c(z)\} \end{aligned}$$

Therefore, $A^c = (\psi_A^c, \chi_A^c)$ is an intuitionistic fuzzy bi ideal of $\mathbb{N}$. ∎

4. Conclusion

In the present paper, by using the idea of fuzzy bi ideals we have pioneered the notion of intuitionistic fuzzy bi ideals of near rings and investigated some of their useful properties. In our view, these definitions and main results can be correspondingly extended to some other algebraic systems.

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