BI-INTERIOR IDEALS OF SEMIGROUPS

M. Murali Krishna Rao

Department of Mathematics
GIT, GITAM University
Visakhapatnam – 530 045, Andhra Pradesh, India

e-mail: mmarapureddy@gmail.com

Abstract

In this paper, as a further generalization of ideals, we introduce the notion of bi-interior ideal as a generalization of quasi ideal, bi-ideal and interior ideal of semigroup and study the properties of bi-interior ideals of semigroup, simple semigroup and regular semigroup.

Keywords: quasi ideal, bi-ideal, interior ideal, bi-interior ideal, bi-quasi ideal, regular semigroup, bi-interior simple semigroup.

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1. Introduction

The notion of ideals was introduced by Dedekind for the theory of algebraic numbers. The notion of ideals was generalized by E. Noether for associative rings. The one and two sided ideals introduced by her, are still central concepts in ring theory. The notion of the bi-ideal in semigroups is a special case of (m, n) ideals introduced by Lajos. In 1952, the concept of bi-ideals for semigroup was introduced by Good and Hughes [1] and the notion of bi-ideals in associative rings was introduced by Lajos and Szasz [4, 5]. We know that the notion of an one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideals and right ideals whereas the bi-ideals are generalization of quasi ideals. Steinfeld [9] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [2] introduced the concept of quasi ideal for a semiring. Murali Krishna Rao [7] introduced bi-quasi-ideals and fuzzy bi-quasi-ideals in Γ-semigroups. Jantanan and Changphas [3] studied 0-minimal $(0, 2)$-bi-ideals. Palakawongna Ayutthaya and Pibaljommee [6] studied ordered quasi k-ideals. In this paper, as a further generalization of ideals, we introduce the notion of bi-interior ideal as a generalization of quasi ideal, bi-ideal and interior ideal of semigroup and study the properties of bi-interior ideals of semigroup.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

A semigroup is an algebraic system $(S,.)$ consisting of a non-empty set S together with an associative binary operation $'' \cdot ''$. A subsemigroup T of S is a non-empty subset T of S such that $TT \subseteq T$. A non-empty subset T of S is called a left (right) ideal of S if $ST \subseteq T (TS \subseteq T)$. A non-empty subset T of S is called an ideal of S if it is both a left ideal and a right ideal of S. A non-empty subset Q of S is called a quasi ideal of S if $QS \cap SQ \subseteq Q$. A subsemigroup T of S is called a bi-ideal of S if $TST \subseteq T$. A subsemigroup T of S is called an interior ideal of S if $STS \subseteq T$. An element a of a semigroup S is called a regular element if there exists an element b of S such that $a = aba$. A semigroup S is called a regular semigroup if every element of S is a regular element. An element $1 \in S$ is said to be unity if $x1 = 1x = x$ for all $x \in S$.

A subsemigroup A of S is called a left(right) bi- quasi ideal of M if $SA \cap ASA(AS \cap ASA) \subseteq A$. A subsemigroup A of S is called a bi- quasi ideal of M if A is a left bi- quasi ideal and a right bi- quasi ideal of M.

A semigroup M is a left (right) simple semigroup if M has no proper left (right) ideal of M. A semigroup M is a bi-quasi simple semigroup if M has no proper bi-quasi ideal of M. A semigroup M is said to be simple semigroup if M has no proper ideals.

3. Bi-interior ideals of semigroups

In this section, we introduce the notion of bi-interior ideal as a generalization of bi-ideal and interior ideal of semigroup and study the properties of bi-interior ideals of semigroups, simple semigroups and regular semigroups.Throghout this paper, M is a semigroup with unity.

Definition 3.1. A non-empty subset B of semigroup M is said to be bi-interior ideal of M if $MBM \cap BMB \subseteq B$.

Definition 3.2. A semigroup M is said to be bi-interior simple semigroup if M has no bi-interior ideals other than M itself.

In the following theorem, we mention some important properties and we omit the proofs since proofs are straight forward.

Theorem 3.3. *Let M be a semigroup. Then the following are hold.*

(1) *Every left ideal is a bi-interior of M.*

(2) *Every right ideal is a bi-interior of M.*

(3) *Every quasi ideal is a bi-interior of M.*

(4) *If L is a bi-interior ideal of semigroup M then $LL \subseteq L$.*

(5) *Intersection of a right ideal and a left ideal of M is a bi-interior ideal of M.*

(6) *If B is a bi-interior ideal of M, then BM and MB are bi-interior ideals of M.*

(7) *If A and B are bi-interior ideals of M, then $A \cap B$ is a bi-interior ideal of M.*

(8) *If B is a bi-interior ideal and T is a subsemigroup of M, then $B \cap T$ is a bi-interior ideal of semigroup M.*

(9) *Every ideal is a bi-interior ideal of M.*

Theorem 3.4. *Let M be a simple semigroup. Every bi-interior ideal is a bi-ideal of M.*

Proof. Let M be a simple semigroup and B be a bi-interior ideal of M. Then $MBM \cap BMB \subseteq B$ and MBM is an ideal of M. Since M is a simple semigroup, we have $MBM = M$. Hence $MBM \cap BMB \subseteq B, \Rightarrow M \cap BMB \subseteq B, \Rightarrow BMB \subseteq B$. Hence the theorem. ∎

Theorem 3.5. *Let M be a semigroup. Then M is a bi-interior simple semigroup if and only if $MaM \cap aMa = M$, for all $a \in M$.*

Proof. Suppose M is a bi-interior simple semigroup and $a \in M$. We know that $MaM \cap aMa$ is a bi-interior ideal of M. Hence $MaM \cap aMa = M$, for all $a \in M$.

Conversely suppose that $MaM \cap aMa = M$, for all $a \in M$. Let B be a bi-interior ideal of semigroup M and $a \in B$. $M = MaM \cap aMa \subseteq MBM \cap BMB \subseteq B$. Therefore $M = B$. Hence M is a bi-interior simple semigroup. ∎

Theorem 3.6. *Let M be a regular semigroup. Then every bi-interior ideal of M is an ideal of M.*

Proof. Let B be a bi-interior ideal of regular semigroup M. Then $BMB \cap MBM \subseteq B$ and $BM \subseteq BMB, BM \subseteq MBM$. Therefore $BM \subseteq BMB \cap MBM \subseteq B$. Similarly, we can show that $MB \subseteq BMB \cap MBM \subseteq B$. Hence the theorem. ∎

Theorem 3.7. *Let M be a semigroup. Then the following statements are equivalent:*

 (i) *M is a bi-interior simple semigroup.*

(ii) *$Ma = M$, for all $a \in M$.*

(iii) *$< a >= M$, for all $a \in M$ and where $< a >$ is the smallest bi-interior ideal generated by a.*

Proof. Let M be a semigroup.

(i)$\Rightarrow$(ii): Suppose M is a bi-interior simple semigroup, $a \in M$ and $B = Ma$. Then B is a left ideal of M. Therefore, by Theorem 3.3, B is a bi-interior ideal of M. Therefore $B = M$. Hence $Ma = M$, for all $a \in M$.

(ii)$\Rightarrow$(iii): Suppose $Ma = M$, for all $a \in M$. Then $Ma \subseteq< a >\subseteq M$, $\Rightarrow M \subseteq< a >\subseteq M$. Therefore $M =< a >$.

(iii)$\Rightarrow$(i): Suppose $< a >$ is the smallest bi-interior ideal of M generated by a, $< a >= M$, A is the bi-interior ideal and $a \in A$. Then $< a >\subseteq A \subseteq M$, $\Rightarrow M \subseteq A \subseteq M$. Therefore $A = M$. Hence M is a bi-interior simple semigroup. $\blacksquare$

Theorem 3.8. *Every bi-ideal of semigroup M is a bi-interior ideal of M.*

Proof. Let B be a bi-ideal of semigroup M. Then $BMB \subseteq B$. Therefore $BMB \cap MBM \subseteq BMB \subseteq B$. Hence every bi-ideal of semigroup M is a bi-interior ideal of M. $\blacksquare$

Theorem 3.9. *Every interior ideal of semigroup M is a bi-interior ideal of M.*

Proof. Let I be an interior ideal of semigroup M. Then $IMI \cap MIM \subseteq MIM \subseteq I$. Hence I is a bi-interior ideal of semigroup M. $\blacksquare$

Theorem 3.10. *If L is a minimal left ideal and R is a minimal right ideal of semigroup M, then $B = RL$ is a minimal bi-interior ideal of M.*

Proof. Obviously $B = RL$ is a bi-interior ideal of M. Suppose A is a bi-interior ideal of semigroup M such that $A \subseteq B$. $MA \subseteq MB = MRL \subseteq L$, since L is a left ideal of M. Similarly, we can prove $AM \subseteq R$. Therefore $MA = L$, $AM = R$. Hence $B = AMML \subseteq AMA.B = RL = RMA \subseteq MA \subseteq MAM$. Therefore $B \subseteq AMA \cap MAM \subseteq A$. Therefore $A = B$. Hence B is a minimal bi-interior ideal of M. $\blacksquare$

Theorem 3.11. *The intersection of a bi-interior ideal B of semigroup M and a subsemigroup A of M is a bi-interior ideal of M.*

Proof. Let B be a bi-interior ideal of semgroup M and A be a subsemigroup of M. Suppose $C = B \cap A$. Then C is a subsemigroup of M, $C \subseteq A$ and $CAC \subseteq AAA \subseteq A \cdots (1)$. $CAC \subseteq BAB \subseteq BMB, MCM \subseteq MBM, \Rightarrow CAC \cap MCM \subseteq BMB \cap MBM \subseteq B, CAC \cap MCM \subseteq CAC \subseteq A$ from (1). Therefore $CAC \cap MCM \subseteq B \cap A = C$. Hence the intersection of a bi-interior ideal B of semigroup M and a subsemigroup A of M is a bi-interior ideal of M. $\blacksquare$

Theorem 3.12. *Let A and C be subsemigroups of semgroup M and $B = AC$. If A is the left ideal then B is a bi-interior ideal of M.*

Proof. Let A and C be subsemgroups of semgroup$M$ and $B = AC$. Suppose A is the left ideal of M. $BMB = ACMAC = ACAC \subseteq AC = B$. Therefore $BMB \cap MBM \subseteq BMB \subseteq B$. Hence B is a bi-interior ideal of M. ∎

Corollary 3.13. *Let A and C be subsemigroups of semigroup M and $B = CA$. If C is a right ideal then B is a bi-interior ideal of M.*

Theorem 3.14. *M is a regular semigroup if and only if $B \cap I \cap L \subseteq BIL$, for any bi-interior ideal B, ideal I and left ideal L of M.*

Proof. Suppose M is a regular semigroup, B, I and L are bi-interior ideal, ideal and left ideal of M respectively. Let $a \in B \cap I \cap L$. Then $a \in aMa$, since M is regular. $a \in aMa \subseteq aMaMa \subseteq BIB \subseteq BMB, a \in aMa \subseteq aMaMa \subseteq MBM, a \in BMB \cap MBM = B$. Hence $B \cap I \cap L \subseteq B$.

Conversely suppose that $B \cap I \cap L \subseteq BIL$, for any bi-interior ideal B, ideal I and left ideal L of M. Let R be a right ideal and L be a left ideal of M. Then by assumption, $R \cap L = R \cap M \cap L \subseteq RML \subseteq RL$. We have $RL \subseteq R$, $RL \subseteq L$. Therefore $RL \subseteq R \cap L$. Hence $R \cap L = RL$. Thus M is a regular semigroup. ∎

Theorem 3.15. *Let M be a semigroup and T be a non-empty subset of M. Then every subsemigroup of T containing $TMT \cup MTM$ is a bi-interior ideal of semigroup M.*

Proof. Let B be a subsemigroup of T containing $TMT \cup MTM$. Then $BMB \subseteq TMT \subseteq TMT \cup MTM \subseteq B$. Therefore $BMB \cap MBM \subseteq B$. Hence B is a bi-interior ideal of M. ∎

Theorem 3.16. *If B is a bi-interior ideal of semigroup M, T is a subsemigroup of M and $T \subseteq B$ then BT is a bi-interior ideal of M.*

Proof. Suppose B is a bi-interior ideal of semigroup M, T is a subsemigroup of M and $T \subseteq B$. Then $(BT)BT \subseteq BBT \subseteq BT$. Hence BT is a subsemigroup of M. We have $MBTM \subseteq MBM$ and $BTMBT \subseteq BMB, \Rightarrow MBTM \cap BTMBT \subseteq MBM \cap BMB \subseteq B. \Rightarrow MBTM \cap BTMBT \subseteq MBM \cap BMB \subseteq B$ and $BT \subseteq B. \Rightarrow MBTM \cap BTMBT \subseteq BT$. Hence BT is a bi-interior ideal of semigroup M. ∎

Corollary 3.17. *Let M be a semigroup and e be an idempotent. Then eM and Me are bi-interior ideals of M respectively.*

Theorem 3.18. *Let B be a bi-ideal of semigroup M and I be an interior ideal of M. Then $B \cap I$ is a bi-interior ideal of M.*

Proof. Suppose B is a bi-ideal of M and I is an interior ideal of M. Obviously $B \cap I$ is a subsemigroup of M. Then $(B \cap I)M(B \cap I) \subseteq BMB \subseteq B$ and $M(B \cap I)M \subseteq MIM \subseteq I$. Therefore $(B \cap I)M(B \cap I) \cap M(B \cap I)M \subseteq B \cap I$. Hence $B \cap I$ is a bi-interior ideal of M. ∎

Theorem 3.19. *Let M be a semigroup and T be a subsemigroup of M. Then every subsemigroup of T containing $MTM \cap TMT$ is a bi-interior ideal of M.*

Proof. Let C be a subsemigroup of T containing $MTM \cap TMT$. Then $MCM \cap CMC \subseteq MTM \cap TMT \subseteq C$. Hence C is a bi-interior ideal of semigroup. ∎

Theorem 3.20. *Let M be a semigroup. If $M = Ma$, for all $a \in M$. Then every bi-interior ideal of M is a quasi ideal of M.*

Proof. Let B be a bi-interior ideal of a semigroup M and $a \in B$. Then $MBM \cap BMB \subseteq B$, $\Rightarrow Ma \subseteq MB$, $\Rightarrow M \subseteq MB \subseteq M$, $\Rightarrow MB = M$, $\Rightarrow BMB = BM$, $\Rightarrow MB \cap BM \subseteq MBM \cap BMB \subseteq B$. Therefore B is a quasi ideal of M. Hence the theorem. ∎

Theorem 3.21. *Let M be a regular semigroup. Then B is a bi-interior ideal of M if and only if $MBM \cap BMB = B$, for all bi-interior ideals B of M.*

Proof. Suppose M is a regular semigroup, B is a bi-interior ideal of M and $x \in B$. Then $MBM \cap BMB \subseteq B$ and there exists $y \in M$, such that $x = xyx \in BMB$. Therefore $x \in MBM \cap BMB$. Hence $MBM \cap BMB = B$.

Conversely suppose that $MBM \cap BMB = M$, for all bi-interior ideals B of M. Let $B = R \cap L$, where R is a right ideal and L is a left ideal of M. Then B is a bi-interior ideal of M. Therefore $MR \cap LM \cap R \cap LMR \cap L = R \cap L$, $R \cap L \subseteq (R \cap L)M(R \cap L) \subseteq RML \subseteq RL \subseteq R \cap L$ since $RL \subseteq L$ and $RL \subseteq R$. Therefore $R \cap L = RL$. Hence M is a regular semigroup. ∎

Theorem 3.22. *If M is a group then M is a bi-interior simple group.*

Proof. Let B be a proper bi-interior ideal of group M, $x \in M$ and $0 \neq a \in B$. Since M is a group, there exists $b \in M$ such that $ab = 1$. Then $abx = x = xab$. Then $x \in BM$. Therefore $M \subseteq BM$. We have $BM \subseteq M$. Hence $M = BM$. Similarly we can prove $M = MB$.

Therefore $BM \subseteq MBM$ and $BM \subseteq BMB$, $\Rightarrow BM \subseteq MBM \cap BMB$, $\Rightarrow M = BM \subseteq MBM \cap BMB \subseteq B$. $\Rightarrow M = B$. Hence group M is a bi-interior simple group. ∎

Theorem 3.23. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ bi-interior ideals of a semigroup M is a bi-interior ideal of M.*

Proof. Let $B = \bigcap_{\lambda \in A} B_\lambda$. Then B is a subsemigroup of M. Since B_λ is a bi-interior ideal of M, we have $B_\lambda M B_\lambda \cap M B_\lambda M \subseteq B_\lambda$, forall $\lambda \in A$, $\Rightarrow (\cap B_\lambda M \cap B_\lambda) \cap M(\cap B_\lambda)M \subseteq (\cap B_\lambda)$, $\Rightarrow (BMB) \cap (MBM) \subseteq B$. Hence B is a bi-interior ideal of M. $\blacksquare$

Theorem 3.24. *Let B be a bi-interior ideal of semigroup M and e be an idempotent such that $eB \subseteq B$. Then eB is a bi-interior ideal of M.*

Proof. Let B be a bi-interior ideal of semigroup M. Suppose $x \in B \cap eM$. Then $x \in B$ and $x = ey, y \in M$. $x = ey = e^2 y = e(ey) = ex \in eB$. Therefore $B \cap eM \subseteq eB$, $eB \subseteq B$ and $eB \subseteq eM$, $\Rightarrow eB \subseteq B \cap eM$, $\Rightarrow eB = B \cap eM$. Hence eB is a bi-interior ideal of M. $\blacksquare$

Theorem 3.25. *Let e and f be idempotents of semigroup M. Then eMf is a bi-interior ideal of M.*

Proof. Suppose e and f be idempotents of semigroup M. Then $eMf \subseteq eM$ and $eMf \subseteq Mf$, $\Rightarrow eMf \subseteq eM \cap Mf$. Let $a \in eM \cap Mf$. Then $a = ec = df$, $c, d \in M$. $a = ec = eec = ea = edf \in eMf$. Therefore $eM \cap Mf \subseteq eMf$. Hence $eM \cap Mf = eMf$. Thus eMf is a bi-interior ideal of semigroup M. $\blacksquare$

Theorem 3.26. *Let M be a semigroup and B be a bi-interior ideal of M. Then B is a minimal bi-interior ideal of M if and only if B is a bi-interior simple subsemigroup of M.*

Proof. Let B be a minimal bi-interior ideal of semigroup M and C be a bi-interior ideal of B. Then $CBC \cap BCB \subseteq C$. Therefore $CBC \cap BCB$ is a bi-interior ideal of M. Since B is a minimal bi-interior ideal of semigroup M, $CBC \cap BCB = B$, $\Rightarrow B = CBC \cap BCB \subseteq C$, $\Rightarrow B = C$.

Conversely suppose that B is a bi-interior simple subsemigroup of M. Let C be a bi-interior ideal of M and $C \subseteq B$. $CBC \cap BCB \subseteq CMC \cap MCM \subseteq C$, $\Rightarrow B = C$. Since B is a bi-interior simple subsemigroup of M. Hence B is a minimal bi-interior ideal of M. $\blacksquare$

Theorem 3.27. *Let M be a semigroup. If B is a bi-interior ideal of M and T is a non-empty subset of B such that BT is a subsemigroup of M then BT is a bi-interior ideal of M.*

Proof. We have $MBM \cap BMB \subseteq B \Rightarrow MBTM \cap BTMBT \subseteq MBM \cap BMB \subseteq B. \Rightarrow MBTM \cap BTMBT \subseteq BT$. Hence BT is a bi-interior ideal of M. $\blacksquare$

Theorem 3.28. *Let B be subsemigroup of a regular semigroup M. Then B can be represented as $B = RL$, where R is a right ideal and L is a left ideal of M if and only if B is a bi-interior ideal of M.*

Proof. Suppose $B = RL$, where R is right ideal of M and L is a left ideal of M. $BMB = RLMRL \subseteq RL, \Rightarrow MBM \cap BMB \subseteq BMB \subseteq RL = B$. Hence B is a bi-interior ideal of M.

Conversely suppose that B is a bi-interior ideal of regular semigroup M. By Theorem 3.22, $MBM \cap BMB = B$. Let $R = BM$ and $L = MB$. Then $R = BM$ is a right ideal of M and $L = MB$ is a left ideal of M. Therefore $BM \cap MB \subseteq MBM \cap BMB = B, \Rightarrow BM \cap MB \subseteq B, \Rightarrow R \cap L \subseteq B$. We have $B \subseteq BM = R$ and $B \subseteq MB = L, \Rightarrow B \subseteq R \cap L, \Rightarrow B = R \cap L = RL$, since M is a regular semigroup.

Hence B can be represented as RL, where R is the right ideal and L is the left ideal of M. Hence the theorem. ∎

Theorem 3.29. *If B is a bi-interior ideal of semigroup M, then B is a bi-quasi ideal of M.*

Proof. Suppose B is a bi-interior ideal of a semigroup M. Then $MBM \cap BMB \subseteq B$. $MB \cap BMB \subseteq MBM \cap BMB \subseteq B$ and $BM \cap BMB \subseteq MBM \cap BMB \subseteq B$. Hence B is a bi-quasi ideal of M. ∎

Theorem 3.30. *If B is a bi-interior ideal of semigroup M and T is a non-empty subset of B such that BT is a subsemigroup of M, then BT is a bi-interior ideal of M.*

Proof. We have $MBM \cap BMB \subseteq B \Rightarrow (MBM \cap BMB)T \subseteq BT, \Rightarrow MBMT \cap BMBT \subseteq BT, \Rightarrow MBTM \cap BTMBT \subseteq MBMT \cap BMBT \subseteq BT$.

Hence BT is a bi-interior ideal of M. ∎

Theorem 3.31. *If semigroup M is a left (right) simple semigroup then every bi-interior ideal of M is a right (left) ideal of M.*

Proof. Let B be a bi-interior ideal of left simple semigroup. Then MB is a left ideal of M and $MB \subseteq M$. Therefore $MB = M$. Then $BM \subseteq MBM$ and $BM = BMB, \Rightarrow BM \subseteq MBM \cap BMB, \Rightarrow BM \subseteq MBM \cap BMB \subseteq B$.

Hence every bi-interior ideal is a right ideal of M. Similarly we can prove for right simple semigroup M. Hence the theorem. ∎

Corollary 3.32. *If semigroup M is a simple semigroup then every bi-interior ideal of M is an ideal of M.*

Theorem 3.33. *Let M be a regular semigroup. If B is a bi-interior ideal of M and B is a regular subsemigroup of M then any bi-interior ideal of B is a bi-interior ideal of M.*

Proof. Let A be a bi-interior ideal of bi-interior ideal B of semigroup M Then by Theorem 3.21, $BAB \cap ABA = A$ and $MBM \cap BMB = B. \cdots (1). \Rightarrow A = BAB \cap ABA \subseteq MAM \cap AMA. \cdots (2)$. From (1) and (2), we get $MAM \cap AMA = A$. Hence A is a bi-interior ideal of M. ∎

4. Conclusion

As a further generalization of ideals, we introduced the notion of bi-interior ideal of semigroup as a generalization of ideal, left ideal, right ideal, bi-ideal,quasi ideal and interior ideal of semigroup and studied some of their properties. We introduced the notion of bi-interior simple semigroup and characterized the bi-interior simple semigroup,the bi-interior regular semigroup using bi-interior ideals of semigroup,We proved every bi-interior ideal of semigroup is a bi-quasi ideal and studied some of the properties of bi-interior ideals of semigroup. In continuity of this paper, we study prime bi-interior ideals, maximal and minimal bi-interior ideals of semigroup.

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