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COMPLICATED BE-ALGEBRAS AND CHARACTERIZATIONS OF IDEALS

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Abstract

In this paper, using the notion of upper sets, we introduced the notions of complicated BE-Algebras and gave some related properties on complicated, self-distributive and commutative BE-algebras. In a self-distributive and complicated BE-algebra, characterizations of ideals are obtained.

Keywords: BE-algebras, complicated BE-algebras, ideals in BE-algebras.2010 Mathematics Subject Classification: 03G25, 06F35.

1. INTRODUCTION

Y. Imai and K. Isėki introduced two classes of abstract algebras called BCKalgebras and BCI-algebras [8, 10]. It is known that the class of BCK-algebras is a proper subclass of BCI-algebras. In [5, 6], Q.P. Hu and X. Li introduced a wide class of abstract algebras called BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of BCH-algebras. J. Neggers and H.S. Kim ([16]) introduced the notion of a d-algebra which is a generalization of BCKalgebras, and also they introduced the notion of B-algebras ([17, 18]). Y.B. Jun, E.H. Roh and H.S. Kim ([11]) introduced a new notion called BH-algebra which is another generalization of BCH/BCI/BCK-algebras. A. Walendziak obtained another equivalent axioms for B-algebras ([20]). C.B. Kim and H.S. Kim ([13]) introduced the notion of BM-algebra which is a specialization of B-algebras. They proved that the class of BM-algebras is a proper subclass of B-algebras and also showed that a BM-algebra is equivalent to a 0-commutative B-algebra. In [14], H.S. Kim and Y.H. Kim introduced the notion of BE-algebra as a generalization of a BCK-algebra. Using the notion of upper sets they gave an equivalent condition of the filter in BE-algebras. In [2] and [3], S.S. Ahn and K.S. So introduced the notion of ideals in BE-algebras, and proved several characterizations of such ideals. Also they generalized the notion of upper sets in BE-algebras and discussed some properties of the characterizations of generalized upper sets related to the structure of ideals in transitive and self distributive BE-algebras. In [4], S.S. Ahn, Y.H. Kim and J.M. Ko are introduced the notion of terminal section of BE-algebras and provided the characterization of a commutative BE-algebras.

B.M. Schein [19] considered systems of the form $(\phi; \circ, \backslash)$, where ϕ is a set of functions closed under the composition " \circ " of functions (and hence $(\phi; \circ)$) is a function semigroup) and the set theoretic subtraction " \backslash " (and hence $(\phi; \backslash)$) is a subtraction algebra in the sence of [1]). B. Zelinka [22] discussed a problem proposed by B.M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y.B. Jun *et al.* [12] introduced the complicated subtraction algebras and investigated several properties on it.

In this paper, using the notion of upper sets, we introduced the notions of complicated BE-Algebras and gave some related properties on complicated, self-distrubutive and commutative BE-algebras. In a self-distributive and complicated BE-algebra, characterizations of ideals are obtained.

2. Preliminaries

Definition 2.1 [14]. An algebra (X; *, 1) of type (2, 0) is called a BE-algebra if, for all a, b, c in X, the following identities hold:

(BE1) a * a = 1, (BE2) a * 1 = 1, (BE3) 1 * a = a, (BE4) a * (b * c) = b * (a * c).

In a BE-algebra X, the relation " \leq " is defined by $a \leq b$ if and only if a * b = 1.

Proposition 2.2 [14]. If (X; *, 1) is a BE-algebra, then

(i) a * (b * a) = 1,

(ii) a * ((a * b) * b) = 1for any $a, b \in X$.

Example 2.1. [14] Let $X = \{1, a, b, c, d, 0\}$ be a set with the following table:

*	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
С	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Then (X; *, 1) is a BE-algebra.

Definition 2.3 [14]. A BE-algebra (X; *, 1) is said to be self-distributive if a * (b * c) = (a * b) * (a * c) for all $a, b, c \in X$.

Example 2.2 [14]. Let $X = \{1, a, b, c, d\}$ be a set with the following table:

*	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c .
c	1	1	b	1	b
d	1	1	1	1	1

Then (X; *, 1) is a self-distributive BE-algebra.

Proposition 2.4 ([2, 4]). Let (X; *, 1) be a self-distributive BE-algebra. If $a \le b$, then, for all a, b, c in X, the following hold:

(i) $c * a \le c * b$, (ii) $b * c \le a * c$, (iii) $a * b \le (b * c) * (a * c)$.

Definition 2.5 [21]. Let X be a BE-algebra. We say that X is commutative if (C) (a * b) * b = (b * a) * a for all $a, b \in X$.

Proposition 2.6 [21]. If (X; *, 1) is a commutative BE-algebra, then for all $a, b \in X$,

$$a * b = 1$$
 and $b * a = 1$ imply $a = b$.

Definition 2.7 [2]. Let X be a BE-algebra. A nonempty subset I of X is called an ideal of X if

(I1) $\forall x \in X \text{ and } \forall a \in I \text{ imply } x * a \in I$,

(I2) $\forall x \in X \text{ and } \forall a, b \in I \text{ imply } (a * (b * x)) * x \in I.$

Corollary 2.8 [2]. Let I be an ideal of X. If $a \in I$ and $a \leq x$, then $x \in I$.

Corollary 2.9 [2]. Let X be a self-distributive BE-algebra. A nonempty subset I of X is an ideal of X if and only if it satisfies the following conditions

(I3) $1 \in I$,

(I4) $x * (y * z) \in I$ and $y \in I$ imply $x * z \in I$ for all $x, y, z \in X$.

3. Complicated BE-algebras

Definition 3.1. Let (X; *, 1) be a BE-algebra and $a, b \in X$. The set

$$A(a,b) = \{x \in X : a * (b * x) = 1\}$$

is called an upper set of a and b. It is easy to see that $1, a, b \in A(a, b)$.

Proposition 3.2. Let (X; *, 1) be a BE-algebra. Then A(a, b) = A(b, a) for all $a, b \in X$.

Proof. It is clear by (BE4).

Example 3.1. Let $X = \{1, a, b, c\}$ be a set with the following table:

*	1	a	b	c
1	1	a	b	c
a	1	1	b	c .
b	1	1	1	c
c	1	1	1	1

It is clear that X is a BE-algebra and $A(1,1) = \{1\}, A(1,a) = A(a,a) = \{1,a\}, A(1,b) = A(a,b) = A(b,b) = \{1,a,b\}$ and A(1,c) = A(a,c) = A(b,c) = A(c,c) = X.

Example 3.2. Let $X = \{1, a, b, c\}$ be a set with the following table:

*	1	a	b	c	
1	1	a	b	c	
a	1	1	b	c	
b	1	a	1	c	
c	1	1	1	1	

It is clear that X is a BE-algebra and $A(1,1) = \{1\}, A(1,a) = A(a,a) = \{1,a\}, A(1,b) = A(b,b) = \{1,b\}, A(a,b) = \{1,a,b\}$ and A(1,c) = A(a,c) = A(b,c) = A(c,c) = X.

Definition 3.3. A BE-algebra (X; *, 1) is called a complicated BE-algebra (c-BE-algebra, shortly) if for all $a, b \in X$, the set A(a, b) has the smallest element. The smallest element of A(a, b) is denoted by $a \otimes b$.

Example 3.3. The BE-algebra X in Example 3.1 is a c-BE-algebra since 1(S)1 = 1, 1(S)a = a, a(S)a = a, 1(S)b = a(S)b = b(S)b = b and 1(S)c = a(S)c = c(S)c = c. But the BE-algebra in Example 3.2 is not a c-BE-algebra since $A(a,b) = \{1,a,b\}$ has no the smallest element.

Proposition 3.4. Let (X; *, 1) be a c-BE-algebra. Then, for all $a, b \in X$,

- (i) $a \otimes b \leq a \text{ and } a \otimes b \leq b$,
- (ii) $a \otimes 1 = a$,
- (iii) $a \otimes b = b \otimes a$,
- (iv) $a \otimes (a * b) \leq b$.

Proof. (i) and (ii) are easily seen by the definition of the c-BE algebra.

(iii) is clear since A(a, b) = A(b, a).

(iv) From Proposition 2.1 (i), since a * ((a * b) * b) = 1, we have $b \in A(a, a * b)$ and hence $a \otimes (a * b) \leq b$.

Proposition 3.5. Let (X; *, 1) be a self-distributive BE-algebra. If, for all $a, b, c \in X$, $a \leq b$ and $b \leq c$ then $a \leq c$.

Proof. Since a * c = 1 * (a * c) = (a * b) * (a * c) = a * (b * c) = a * 1 = 1, we have $a \le c$.

Proposition 3.6. Let (X; *, 1) be a self-distributive c-BE-algebra. Then, for all $a, b, c \in X$,

- (i) $a \leq b$ implies $a \otimes c \leq b \otimes c$,
- (ii) $(a * b) \circledast (b * c) \le a * c.$

Proof. (i) Let $a \leq b$. Since X is self-distributive, by Proposition 2.4 (ii), we have $b * (b \otimes c) \leq a * (b \otimes c)$. Also since $b \otimes c \in A(b, c)$, we have $c \leq b * (b \otimes c)$. Then by Proposition 3.5, we get $c \leq a * (b \otimes c)$. Hence we obtain $b \otimes c \in A(a, c)$ and $a \otimes c \leq b \otimes c$.

(ii) By Proposition 2.4. (iii), we have $a * b \le (b * c) * (a * c)$. Hence we see that $a * c \in A(a * b, b * c)$ and $(a * b) \circledast (b * c) \le a * c$.

Theorem 3.7. Let (X; *, 1) be a self-distributive and commutative c-BE-algebra. Then $(X; \circledast)$ is a commutative monoid.

Proof. By Proposition 3.4 (ii) and (iii), we need only to show that $(X; \circledast)$ is associative. Say $(a \circledast b) \circledast c = u$. Then, since $u \in A(a \circledast b, c)$ and $A(a \circledast b, c) = A(c, a \circledast b)$, we know that

$$(3.1) a \circledast b \le c * u$$

and

$$(3.2) c \le (a \otimes b) * u.$$

Hence using the equation (3.1), we have, by Proposition 2.4 (i) and (BE4),

(3.3)
$$b * (a \otimes b) \le b * (c * u) = c * (b * u).$$

Since $a \leq b * (a \otimes b)$, using the equation (3.3) and Proposition 3.5, we obtain

$$(3.4) a \le c * (b * u).$$

From the equation (3.4), we have $b * u \in A(a, c)$ and $a \otimes c \leq b * u$. So we see that $u \in A(a \otimes c, b)$, that is,

$$(3.5) (a \otimes c) \otimes b \le (a \otimes b) \otimes c = u.$$

Since the equation (3.5) is true for all $a, b, c \in X$, the following inequality is true:

$$(3.6) (a \$b) \$c \le (a \$c) \$b.$$

Hence by Proposition 2.6, using the equation (3.5) and (3.6), we get

$$(3.7) (a \otimes b) \otimes c = (a \otimes c) \otimes b.$$

Then we obtain $(a \otimes b) \otimes c = (b \otimes a) \otimes c = (b \otimes c) \otimes a = a \otimes (b \otimes c)$.

Proposition 3.8. If (X; *, 1) is a self-distributive and commutative c-BE-algebra and $X \neq \{1\}$, then $(X; \circledast)$ has no group structure.

Proof. Let $1 \neq a \in X$. Hence we have $a \leq 1$. If there exists an element $b \in X$ such that $a(\underline{S}b = b(\underline{S}a = 1)$, then since $1 = a(\underline{S}b \leq a \leq 1)$, we have a = 1. This is a contradiction.

Proposition 3.9. Let (X; *, 1) be a self-distributive and commutative c-BEalgebra. Then $a \leq b$ implies $a \otimes b = a$.

Proof. (i) Let $a \leq b$. Hence we have a * b = 1. Then we get

$$a * (a \otimes b) = 1 * (a * (a \otimes b))$$

= (a * b) * (a * (a \otimes b))
= a * (b * (a \otimes b)), by self-distributivity property
= 1

since $a \otimes b \in A(a, b)$. Hence $a \leq b * (a \otimes b)$. Then we have $a \leq a \otimes b$. Also we know that $a \otimes b \leq a$. Hence we obtain $a \otimes b = a$ by Proposition 2.6.

Now, in a c-BE-algebra, define the set

(3.8)
$$B(a,b) = \{x \in X : x \otimes a \le b\}$$

Theorem 3.10. Let (X; *, 1) is a self-distributive c-BE-algebra. Then the set B(a, b) in equation (3.8) has the greatest element and it is a * b.

Proof. Since $a * b \le a * b$, we have $b \in A(a * b, a)$. Hence we get $(a * b)(\underline{S}a \le b$. So, it is seen that $a * b \in B(a, b)$. If $c \in B(a, b)$, we write $c(\underline{S}a \le b)$. By Proposition 2.4 (i), we have $a * (c(\underline{S}a) \le a * b)$. Since $c(\underline{S}a \in A(c, a))$, we have $c \le a * (c(\underline{S}a))$. Then we obtain $c \le a * b$, by Proposition 3.5. Hence a * b is the greatest element of B(a, b).

Proposition 3.11. Let (X; *, 1) be a self-distributive and commutative c-BEalgebra. Then

- (i) $a \otimes b \leq a * b \leq (a \otimes c) * (c \otimes b)$,
- (ii) (a * b) (a * b) (a * b)
- (iii) $(a \otimes b) * c = a * (b * c),$
- (iv) $a * (b \otimes c) = (a * b) \otimes (a * c),$
- (v) a \$b is the greatest lower bound of the set $\{a, b\}$.

Proof. (i) Using Proposition 3.4 (iv) and Proposition 3.6 (i), we have $c(a(a(a * b)) \le c(a))$. We get $(c(a)(a * b)) \le c(a)$ or by Proposition 3.4 (iii), $(a * b)(a(a)) \le c(a) \le c(a)$. Hence since $a * b \in B(c(a)(a))$, we obtain $a * b \le (a(a)) * (c(a))$. Also it is known that $a(a) \le b \le a * b$. By Proposition 3.5, we get $a(a) \le a * b \le (a(a)) * (c(a))$.

(ii) Since $a * b \in B(a, b)$, we have $(a * b) \otimes a \leq b$. Using Proposition 3.4 (i), commutativity and associativity of the operation \otimes , we get $(a*b) \otimes (a \otimes a) \leq a \otimes b$. By Proposition 3.9, we see that $a \otimes a = a$ since $a \leq a$. Hence $(a * b) \otimes a \leq a \otimes b$. Secondly, since $b \le a * b$, by commutativity of the operation (S) and Proposition 3.6 (i), we have $a \otimes b \le (a * b) \otimes a$. So we obtain $(a * b) \otimes a = a \otimes b$ by Proposition 2.6.

(iii) $a \otimes b \in A(a, b)$ implies $a \leq b * (a \otimes b)$. Also from Proposition 2.4 (iii), we have $b * (a \otimes b) \leq ((a \otimes b) * c) * (b * c)$. So we get $a \leq ((a \otimes b) * c) * (b * c)$ by Proposition 3.5. Then we have $(a \otimes b) * c \leq a * (b * c)$). Secondly, using Proposition 2.2 (ii), Proposition 2.4 (iii) and (BE4), since

$$b \le (b * c) * c \le (a * (b * c)) * (a * c) = a * ((a * (b * c)) * c),$$

we have $b \leq a * ((a * (b * c)) * c)$ or $a \leq b * ((a * (b * c)) * c)$. Then we obtain $a \otimes b \leq (a * (b * c)) * c$ or $a * (b * c) \leq (a \otimes b) * c$. Consequently we see that $a * (b * c) = (a \otimes b) * c$.

(iv) By (i), we have $a * c \le (a \otimes b) * (b \otimes c)$ or $a \otimes b \le (a * c) * (b \otimes c)$. Then we get $a * (a \otimes b) \le a * ((a * c) * (b \otimes c))$ by Proposition 2.4 (i). We can write $a * b \le (a \otimes a) * (a \otimes b) \le (a * c) * (a * (b \otimes c))$ by (i). Hence since $a * (b \otimes c) \in A(a * b, a * c)$, we have

$$(3.9) \qquad (a*b) \widehat{\mathbb{S}}(a*c) \le a*(b \widehat{\mathbb{S}}c).$$

Secondly, since $b \otimes c \leq b$, we have $a * (b \otimes c) \leq a * b$ by Proposition 2.4 (i). Hence we get

$$(3.10) \qquad (a*(b \otimes c)) \otimes (a*c) \le (a*b) \otimes (a*c).$$

Also since $b \otimes c \leq c$, we have $a * (b \otimes c) \leq a * c$ and so we get $(a * (b \otimes c)) \otimes (a * (b \otimes c)) \leq (a * (b \otimes c)) \otimes (a * c)$, that is

$$(3.11) a * (b \otimes c) \le (a * (b \otimes c)) \otimes (a * c).$$

Hence from the equation (3.10) and (3.11) and by Proposition 3.5, we obtain

$$(3.12) a * (b \otimes c) \le (a * (b \otimes c)) \otimes (a * c).$$

The equations (3.9) and (3.12) show that $a * (b \otimes c) = (a * (b \otimes c)) \otimes (a * c)$ by Proposition 2.6.

(v) Since $a \otimes b \leq a$ and $a \otimes b \leq b$, $a \otimes b$ is the lower bound of the set $\{a, b\}$. Let c be another lower bound of the set $\{a, b\}$. Then we know that c * a = 1 and c * b = 1. So since $c * (a \otimes b) = (c * a) \otimes (c * b) = 1 \otimes 1 = 1$, we have $c \leq a \otimes b$. **Remark 3.1.** Let (X; *, 1) be a BE-algebra. In [21], the binary operation "+" on X was defined as the following: for any $a, b \in X$,

$$a + b = (a * b) * b.$$

Also the author proved that if (X; *, 1) is a commutative BE-algebra, then (X; +) is a semilattice. By Proposition 3.11 (v), we proved that a self-distributive and commutative c-BE-algebra X is a semilattice under the operation "(S)". In a self-distributive and commutative c-BE-algebra, since $a \leq a + b$ by Proposition 2.2 (ii) and using Proposition 3.9, we see that a(S(a + b) = a). Also, since $a \leq b$ implies a + b = b and since $a(S)b \leq a$, we have (a(S)b) + a = a. Therefore any self-distributive and commutative c-BE-algebra is a lattice with respect to operations "(S)" and "+".

Now we provide characterizations of ideals in a self-distributive c-BE-algebra.

Corollary 3.12 [2]. Let (X; *, 1) be a self-distributive BE-algebra. A nonempty subset I of X is an ideal of X if and only if $A(u, v) \subseteq I$ for all $u, v \in I$.

Theorem 3.13. Let (X; *, 1) be a self-distributive c-BE-algebra. A nonempty subset I of X is an ideal of X if and only if it satisfies the following conditions:

- (i) $\forall a \in I, \forall x \in X, a \leq x \Longrightarrow x \in I$,
- (ii) $\forall a, b \in I, \exists c \in I, c \leq a \text{ and } c \leq b.$

Proof. Let I be an ideal of X. (i) follows from the Corollary 2.8. Let $a, b \in I$. From Corollary 3.12, we have $A(a, b) \subseteq I$. Then we get $a \otimes b \in I$. If we take $a \otimes b = c$, then we have $c \leq a$ and $c \leq b$ by Proposition 3.4 (i) which proves (ii). Conversely, let I be a non-empty subset of X satisfying (i) and (ii). Since for $a \in I, a \leq 1$ by (BE2), we have $1 \in I$ by (i). For any $a, b, c \in X$, let $b \in I$ and $a * (b * c) \in I$. By (ii), there exists $d \in I$ such that $d \leq b$ and $d \leq a * (b * c)$. Then using (BE3), (BE4), and self-distributivity, we have

$$1 = d * (a * (b * c)) = d * (b * (a * c)) = (d * b) * (d * (a * c)) = d * (a * c).$$

Hence, we get $d \le a * c$. By (i), it is obtained $a * c \in I$. So I is an ideal of X by Corollary 2.9.

Theorem 3.14. Let (X; *, 1) be a self-distributive c-BE-algebra. A non-empty subset I of X is an ideal of X if and only if it satisfies the following conditions:

- (i) $\forall a \in I, \forall x \in X, a \leq x \Longrightarrow x \in I$,
- (ii) $\forall a, b \in I, a \otimes b \in I$.

Proof. The necessity is given in the proof of Theorem 3.13. Conversely, since for $a \in I, a \leq 1$ by (BE2), we have $1 \in I$ by (i). Let I be a non-empty subset of X satisfying (i) and (ii). We know that $x * y \in B(x, y)$ in a self-distributive c-BE-algebra. So (x * y) (S) $x \leq y$ and hence

$$(3.13) x(S)(x*y) \le y.$$

Now let $y \in I$ and $x * (y * z) \in I$. By (ii) and (BE4), we get $y(s)(x * (y * z)) = y(s)(y * (x * z)) \in I$. From the equation 3.13, it is clear that $y(s)(y * (x * z)) \leq x * z$. Hence it is obtained $x * z \in I$ by (i). Consequently, I is an ideal of X by Corollary 2.9.

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