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APPLICATIONS OF MAXIMAL μ-OPEN SETS IN GENERALIZED TOPOLOGY AND QUASI TOPOLOGY

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Abstract

In this paper, some fundamental properties of maximal μ-open sets such as decomposition theorem for a maximal μ-open set, are given in a generalized topological space. Some basic properties of intersection of maximal μ-open sets are established, cohere the law of μ-radical μ-closure in a quasi topological space is obtained, among the other things.

Keywords: μ-open set, maximal μ-open set, μ-radical.

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1. Introduction

For the last one decade or so, a new area of study has emerged and has been rapidly growing. The area is concerned with the investigations of generalized topological spaces and several classes of generalized types of open sets. On the other hand, some properties of maximal open sets and minimal closed sets in a topological space have been studied in [6, 7]. Our aim here is to study the notion of maximal μ-open and minimal μ-closed sets by using the concept of generalized topology introduced by Á. Császár [2]. We first recall some definitions given in [2]. Let X be a non-empty set and $expX$ denote the power set of X. We call a class $\mu \subseteqq expX$ a generalized topology (briefly, GT) [2], if $\emptyset \in \mu$ and unions of elements of μ belong to μ. A set X, with a GT μ on it is said to be a generalized topological space (briefly, GTS) and is denoted by (X, μ). A GT μ is said to be a quasi topology (briefly QT) [3, 5] if $M, M^{'} \in \mu$ implies $M \cap M^{'} \in \mu$. The pair (X, μ) is said to be a QTS if μ is a QT on X. For a GTS (X, μ), the elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets. For $A \subseteqq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A (see [2, 3]).

It is easy to observe that i_μ and c_μ are idempotent and monotonic, where $\gamma : \exp X \to \exp X$ is said to be idempotent iff for each $A \subseteqq X$, $\gamma(\gamma(A)) = \gamma(A)$, and monotonic iff $\gamma(A) \subseteqq \gamma(B)$ whenever $A \subseteqq B \subseteqq X$. It is also well known from [1, 3] that if μ is a GT on X and $A \subseteqq X$, $x \in X$, then $x \in c_\mu(A)$ iff $(x \in M \in \mu \Rightarrow M \cap A \neq \emptyset)$ and that $c_\mu(X \setminus A) = X \setminus i_\mu(A)$.

The purpose of this paper is to study some interesting properties of μ-radicals of maximal μ-open sets. We then introduce a useful decomposition theorem for maximal μ-open sets, this has been used later to describe a sufficient condition for maximal μ-open sets. Finally, the μ-closures of the μ-radicals of maximal μ-open sets are considered to establish "Law of μ-radical μ-closure".

2. Fundamental properties of μ-radicals

Definition 1 [8]**.** A proper nonempty μ-open set A of a GTS (X, μ) is called a *maximal μ-open set* if there is no μ-open set strictly between A and X.

Theorem 2 [8]**.** *Let A, B be two maximal μ-open sets in a GTS (X, μ). Then either $A \cup B = X$ or $A = B$.*

Definition 3. Let $\mathcal{U} = \{U_\lambda : \lambda \in \Lambda\}$ be a collection of some maximal μ-open sets in a GTS (X, μ). Then we call $\cap\,\mathcal{U} = \cap_{\lambda \in \Lambda} U_\lambda$ the *μ-radical of $\mathcal{U}$*.

The intersection of maximal ideals of a ring $\mathcal{R}$ is known as the *radical* of the ring $\mathcal{R}$ [4]. Following the terminology of the theory of rings, the terminology "*μ-radical*" has been introduced here.

Theorem 4. *Let (X, μ) be a GTS and U_λ be a maximal μ-open set for any element λ of Λ with $|\Lambda| \geqq 2$ and $U_\lambda \neq U_\alpha$ for any $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. Then,*

(i) $X \setminus \cap_{\lambda \in \Lambda \setminus \{\alpha\}} U_\lambda \subseteqq U_\alpha$, *for any* $\alpha \in \Lambda$.

(ii) $\cap_{\lambda \in \Lambda \setminus \{\alpha\}} U_\lambda \neq \emptyset$ *for any* $\alpha \in \Lambda$.

Proof. (i) Let α be any element of Λ. Then by Theorem 2, we have $X \setminus U_\alpha \subseteqq U_\lambda$ for any element λ of Λ with $\lambda \neq \alpha$. Hence $X \setminus U_\alpha \subseteqq \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$. Thus, $X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda \subseteqq U_\alpha$.

(ii) If $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda = \emptyset$ then by (i) above, $X = U_\alpha$. But this is a contradiction to the fact that U_α is a maximal μ-open set. Therefore $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda \neq \emptyset$. ■

Corollary 5. *Let U_λ be a maximal μ-open set for each element λ of Λ in a GTS (X, μ) and $U_\lambda \neq U_\alpha$ for any elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. If $|\Lambda| \geq 3$, then $U_\lambda \cap U_\alpha \neq \emptyset$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$.*

Proof. The proof follows from Theorem 4(ii). ■

Theorem 6. *Let (X, μ) be a GTS and let U_λ be a maximal μ-open set for any $\lambda \in \Lambda$ with $|\Lambda| \geq 2$ and $U_\lambda \neq U_\alpha$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. Then $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda \nsubseteqq U_\alpha \nsubseteqq \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$ for any $\alpha \in \Lambda$.*

Proof. Let α be any element of Λ. If $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda \subseteqq U_\alpha$, then $X = (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \cup (\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \subseteqq U_\alpha$ (by Theorem 4) i.e., $U_\alpha = X$. This contradicts the fact that U_α is maximal μ-open. Now if $U_\alpha \subseteqq \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$, then we have $U_\alpha \subseteqq U_\lambda$ for each $\lambda \in \Lambda \setminus \{\alpha\}$ and hence $U_\alpha = U_\lambda$ for any $\lambda \in \Lambda \setminus \{\alpha\}$ (as U_α is maximal μ-open). This is again a contradiction to the fact that $U_\lambda \neq U_\alpha$ for $\lambda \neq \alpha$. Hence the theorem. ■

Corollary 7. *Let (X, μ) be a GTS and U_λ be a maximal μ-open set for each $\lambda \in \Lambda$ and $U_\lambda \neq U_\alpha$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. If $(\emptyset \neq)\ \Delta \subsetneqq \Lambda$, then $\cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda \nsubseteqq \cap_{\alpha\in\Delta} U_\alpha \nsubseteqq \cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda$.*

Proof. For $\alpha \in \Delta$, we see that $\cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda = \cap_{\lambda\in((\Lambda\setminus\Delta)\cup\{\alpha\})\setminus\{\alpha\}} U_\lambda \nsubseteqq U_\alpha$ (by Theorem 6). Thus $\cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda \nsubseteqq \cap_{\alpha\in\Delta} U_\alpha$. Again, $\cap_{\alpha\in\Delta} U_\alpha = \cap_{\alpha\in\Lambda\setminus(\Lambda\setminus\Delta)} U_\alpha \nsubseteqq \cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda$ and hence $\cap_{\alpha\in\Delta} U_\alpha \nsubseteqq \cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda$. ■

Theorem 8. *Let for each $\lambda \in \Lambda$, U_λ be a maximal μ-open set in a GTS (X, μ) and $U_\lambda \neq U_\alpha$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. If $(\emptyset \neq)\ \Delta \subsetneqq \Lambda$, then $\cap_{\lambda\in\Lambda} U_\lambda \subsetneqq \cap_{\alpha\in\Delta} U_\alpha$.*

Proof. We observe that $\cap_{\lambda\in\Lambda} U_\lambda = (\cap_{\lambda\in\Lambda\setminus\Delta} U_\lambda) \cap (\cap_{\alpha\in\Delta} U_\alpha) \subsetneqq \cap_{\alpha\in\Delta} U_\alpha$ (by Corollary 7). ■

Theorem 9 (Decomposition Theorem)**.** *Let (X, μ) be a GTS and U_λ be a maximal μ-open set for $\lambda \in \Lambda$, where $|\Lambda| \geqq 2$ and $U_\lambda \neq U_\alpha$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. Then for any $\alpha \in \Lambda$, $U_\alpha = (\cap_{\lambda\in\Lambda} U_\lambda) \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda)$.*

Proof. $(\cap_{\lambda\in\Lambda} U_\lambda) \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) = ((\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \cap U_\alpha) \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) = ((\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda)) \cap (U_\alpha \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda)) = X \cap [U_\alpha \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda)] = U_\alpha \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) = U_\alpha$ (by Theorem 4). ■

As an application of Theorem 9, we give another proof of Theorem 8.

Proof. Since $\emptyset \neq \Delta \subsetneqq \Lambda$, there exists an element ν of Λ such that $\nu \notin \Delta$ and an element α of Δ. If $|\Delta| = 1$, then we have $\cap_{\lambda\in\Lambda} U_\lambda \subseteqq U_\alpha$. If $\cap_{\lambda\in\Lambda} U_\lambda = U_\alpha$, then we have $U_\alpha \subseteqq U_\lambda$ for any element λ of Λ. Since U_λ is a maximal μ-open set for any element λ of Λ, we have $U_\alpha = U_\lambda$, which contradicts our assumption. Hence, we have $\cap_{\lambda\in\Lambda} U_\lambda \subsetneqq U_\alpha$. If $|\Delta| \geq 2$, then by Theorem 9, we have

$$\begin{aligned} U_\nu &= (\cap_{\lambda\in\Lambda} U_\lambda) \cup (X \setminus \cap_{\lambda\in\Lambda\setminus\{\nu\}} U_\lambda), \\ U_\alpha &= (\cap_{\delta\in\Delta} U_\delta) \cup (X \setminus \cap_{\delta\in\Delta\setminus\{\alpha\}} U_\delta). \end{aligned}$$

If $\cap_{\lambda\in\Lambda} U_\lambda = \cap_{\delta\in\Delta} U_\delta$, then $\cap_{\delta\in\Delta} U_\delta = \cap_{\lambda\in\Lambda} U_\lambda \subseteqq \cap_{\lambda\in\Lambda\setminus\{\nu\}} U_\lambda \subseteqq \cap_{\delta\in\Delta} U_\delta$. Hence, we have $\cap_{\lambda\in\Lambda\setminus\{\nu\}} U_\lambda = \cap_{\delta\in\Delta} U_\delta$. Therefore, $\cap_{\lambda\in\Lambda\setminus\{\nu\}} U_\lambda = \cap_{\delta\in\Delta} U_\delta \subseteqq \cap_{\delta\in\Delta\setminus\{\alpha\}} U_\delta$. Hence, we see that $U_\nu \supseteqq U_\alpha$. It follows that $U_\nu = U_\alpha$ with $\nu \neq \alpha$. This contradicts our assumption. ∎

The next theorem gives a description of maximal μ-open sets.

Theorem 10. *Let (X, μ) be a GTS and U_λ be a maximal μ-open set for each $\lambda \in \Lambda$ with $|\Lambda| \geqq 2$ and $U_\lambda \neq U_\alpha$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. If $\cap_{\lambda\in\Lambda} U_\lambda = \emptyset$, then $\{U_\lambda : \lambda \in \Lambda\}$ is the set of all maximal μ-open sets of X.*

Proof. If possible, let U_ν be another maximal μ-open set of X which is not equal to U_λ for any $\lambda \in \Lambda$. Then $\emptyset = \cap_{\lambda\in\Lambda} U_\lambda = \cap_{\lambda\in(\Lambda\cup\{\nu\})\setminus\{\nu\}} U_\lambda$. By Theorem 4, we see that $\cap_{\lambda\in(\Lambda\cup\{\nu\})\setminus\{\nu\}} U_\lambda \neq \emptyset$. This contradicts our assumption. ∎

Example 11. Let (X, μ) be a GTS such that for each $x \in X$, $\{x\}$ is μ-closed. Then $X \setminus \{a\}$ is a maximal μ-open set for any $a \in X$. Since $\cap\{X \setminus \{a\} : a \in X\} = \emptyset$, by Theorem 10 it follows that $\{X \setminus \{a\} : a \in X\}$ is the set of all maximal μ-open sets of X.

3. μ-RADICAL μ-CLOSURE IN QT

Proposition 12. *Let μ be a QT and A, B be two subsets of X. If $A \cup B = X$, $A \cap B$ is a μ-closed set and A is a μ-open set, then B is a μ-closed set.*

Proof. Since $X \setminus A \subseteqq B$, we have $(A \cap B) \cup (X \setminus A) = (A \cup (X \setminus A)) \cap (B \cup (X \setminus A)) = B \cup (X \setminus A) = B$. Since $A \cap B$ and $X \setminus A$ are μ-closed sets and μ is a QT, B is a μ-closed set. ∎

Proposition 13. *Let μ be a QT. Let U_λ be a μ-open set for each element λ of Λ and $U_\lambda \cup U_\alpha = X$ for any two elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. If $\cap_{\lambda\in\Lambda} U_\lambda$ is a μ-closed set, then $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$ is a μ-closed set for any element α of Λ.*

Proof. Let α be any element of Λ. Since $U_\lambda \cup U_\alpha = X$ for any element λ of Λ with $\lambda \neq \alpha$, $U_\alpha \cup (\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) = \cap_{\lambda\in\Lambda\setminus\{\alpha\}}(U_\alpha \cup U_\lambda) = X$. Since $U_\alpha \cap (\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) = \cap_{\lambda\in\Lambda} U_\lambda$ is a μ-closed set, by Proposition 12, $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$ is μ-closed for any element α of Λ. ■

Theorem 14. *Let μ be a QT on X and U_λ be a maximal μ-open set for each element $\lambda \in \Lambda$. If $U_\lambda \neq U_\alpha$ for $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$ and $\cap_{\lambda\in\Lambda} U_\lambda$ is a μ-closed set, then $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$ is a μ-closed set for any element α of Λ.*

Proof. By Theorem 2, we have $U_\lambda \cup U_\alpha = X$ for any λ and α of Λ with $\lambda \neq \alpha$. Thus by Theorem 13, we have $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$ is a μ-closed set. ■

The last three results are false in a GT follows from the next example.

Example 15. Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Then (X, μ) is a GTS. Now $A = \{a, b\}$ and $B = \{b, c\}$ are two maximal μ-open sets in X such that $A \cap B = \{b\}$ is μ-closed but A is not a μ-closed set.

Theorem 16 [3]**.** *If μ be a QT on X then c_μ is μ-friendly i.e., for $A \subseteqq X$ and $M \in \mu$, $c_\mu(A) \cap M \subseteqq c_\mu(A \cap M)$.*

We also recall from [8] that for any maximal μ-open set A in GTS (X, μ), either $c_\mu(A) = X$ or $c_\mu(A) = A$.

Theorem 17. *Let U_λ be a maximal μ-open set in a QTS (X, μ) for each element λ of a finite set Λ. If $c_\mu(\cap_{\lambda\in\Lambda} U_\lambda) \neq X$, then there exists an element λ of Λ such that $c_\mu(U_\lambda) = U_\lambda$.*

Proof. Assume that $c_\mu(U_\lambda) = X$ for each element $\lambda \in \Lambda$. Let α be any element of Λ. Since $\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\alpha$ is a μ-open set, we have $c_\mu(\cap_{\lambda\in\Lambda} U_\lambda) = c_\mu((\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \cap U_\alpha) \supseteqq (\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \cap c_\mu(U_\alpha)$ (by Theorem 16)$=(\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \cap X = \cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda$. Hence, $c_\mu(\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \subseteqq c_\mu(\cap_{\lambda\in\Lambda} U_\lambda)$. On the other hand as c_μ is an increasing operator we have, $c_\mu(\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) \supseteqq c_\mu(\cap_{\lambda\in\Lambda} U_\lambda)$. Thus it follows that $c_\mu(\cap_{\lambda\in\Lambda\setminus\{\alpha\}} U_\lambda) = c_\mu(\cap_{\lambda\in\Lambda} U_\lambda)$. Then by induction on the elements of Λ, we see that $c_\mu(\cap_{\lambda\in\Lambda} U_\lambda) = c_\mu(U_\lambda) = X$ for any element λ of Λ. This contradicts our assumption that $c_\mu(\cap_{\lambda\in\Lambda} U_\lambda) \neq X$. Thus we see that there exists an element λ of Λ such that $c_\mu(U_\lambda) = U_\lambda$. ■

The above theorem is not necessarily true when Λ is an infinite set, as shown by the following example. Also by another example we show that the above theorem is false in a GTS.

Example 18. (a) Let $\mathbb{R}$ denotes the real line with μ as usual topology. Let $U_x = \mathbb{R}\setminus\{x\}$ for any $x \in \mathbb{R}$. Then by Theorem 10, $\{U_x : x \in \mathbb{R}\}$ is the collection of all maximal μ-open sets and $c_\mu(\cap_{x\in\mathbb{R}}U_x) = c_\mu(\emptyset) = \emptyset \neq \mathbb{R}$. However, $c_\mu(U_x) = \mathbb{R}$ for each element $x \in \mathbb{R}$.

(b) Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Then (X, μ) is a GTS. Let $A_1 = \{a, b\}$ and $A_2 = \{b, c\}$ and $A_3 = \{a, c\}$. Then A_1, A_2 and A_3 are three maximal μ-open sets in X. It is easy to see that $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) \neq X$ but $c_\mu(U_\lambda) \neq U_\lambda$ for any λ of Λ.

The μ-radicals of maximal μ-open sets have the following outstanding property.

Theorem 19 (Law of μ-radical μ-closure)**.** *Let Λ be a finite set and U_λ be a maximal μ-open set for each λ of Λ in a QTS (X, μ). Let Δ be a subset of Λ such that $c_\mu(U_\lambda) = U_\lambda$ for any $\lambda \in \Delta$ and $c_\mu(U_\lambda) = X$ for any $\lambda \in \Lambda \setminus \Delta$. Then $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) = \cap_{\lambda\in\Delta}U_\lambda (= X$ if $\Delta = \emptyset)$.*

Proof. If $\Delta = \emptyset$, then the result follows from Theorem 17. If $\Delta \neq \emptyset$, $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) = c_\mu((\cap_{\lambda\in\Delta}U_\lambda) \cap (\cap_{\lambda\in\Lambda\setminus\Delta}U_\lambda)) \supseteqq (\cap_{\lambda\in\Delta}U_\lambda) \cap c_\mu(\cap_{\lambda\in\Lambda\setminus\Delta}U_\lambda) = \cap_{\lambda\in\Delta}U_\lambda \cap X = \cap_{\lambda\in\Delta}U_\lambda$ by Theorem 16 and the fact that $\cap_{\lambda\in\Delta}U_\lambda$ is a μ-open set. Thus, $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) = c_\mu(c_\mu(\cap_{\lambda\in\Lambda}U_\lambda)) \supseteqq c_\mu(\cap_{\lambda\in\Delta}U_\lambda)$. Again as $\cap_{\lambda\in\Lambda}U_\lambda \subseteqq \cap_{\lambda\in\Delta}U_\lambda$, we have $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) \subseteqq c_\mu(\cap_{\lambda\in\Delta}U_\lambda)$. Thus $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) = c_\mu(\cap_{\lambda\in\Delta}U_\lambda)$. The μ-radical $\cap_{\lambda\in\Delta}U_\lambda$ is a μ-closed set since U_λ is μ-closed for any $\lambda \in \Delta$ by our assumption. Thus we have, $c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) = \cap_{\lambda\in\Delta}U_\lambda$. ■

As an application of Theorem 19, we prove the next theorem.

Theorem 20. *Let μ be a QT on X and let U_λ be a maximal μ-open set for each element λ of a finite set Λ and $U_\lambda \neq U_\alpha$ for any elements $\lambda, \alpha \in \Lambda$ with $\lambda \neq \alpha$. If $\cap_{\lambda\in\Lambda}U_\lambda$ is a μ-closed set then U_λ is μ-closed for each element λ of Λ.*

Proof. Let Δ be a subset of Λ such that $c_\mu(U_\lambda) = U_\lambda$ for any $\lambda \in \Delta$ and $c_\mu(U_\lambda) = X$ for any $\lambda \in \Lambda \setminus \Delta$. By hypothesis, the μ-radical $\cap_{\lambda\in\Lambda}U_\lambda$ is a μ-closed set. By Theorem 19, we can say that $\Delta \neq \emptyset$. Then for $\Delta \subseteqq \Lambda$, $\cap_{\lambda\in\Lambda}U_\lambda = c_\mu(\cap_{\lambda\in\Lambda}U_\lambda) = \cap_{\lambda\in\Delta}U_\lambda$ (by Theorem 19). Thus by Theorem 8, $\Lambda = \Delta$. ■

That the theorem is not true in a generalized topological space is shown in the next example.

Example 21. We note that (X, μ) is a GTS where X and μ are as given in Example 18(b). Let $A_1 = \{a, b\}$ and $A_2 = \{b, c\}$ and $A_3 = \{a, c\}$. Then A_1, A_2 and A_3 are three maximal μ-open sets in X such that $\cap\{A_i : i = 1, 2, 3\} = \emptyset$ is μ-closed but A_i is not μ-closed for $i = 1, 2, 3$.

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