

## CLIFFORD CONGRUENCES ON GENERALIZED QUASI-ORTHODOX GV-SEMIGROUPS

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### Abstract

A semigroup  $S$  is said to be completely  $\pi$ -regular if for any  $a \in S$  there exists a positive integer  $n$  such that  $a^n$  is completely regular. A completely  $\pi$ -regular semigroup  $S$  is said to be a GV-semigroup if all the regular elements of  $S$  are completely regular. The present paper is devoted to the study of generalized quasi-orthodox GV-semigroups and least Clifford congruences on them.

**Keywords:** Clifford semigroup, Clifford congruence, generalized quasi-orthodox semigroup.

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### 1. INTRODUCTION

The study of the structure of semigroups is essentially influenced by the study of the congruences defined on them. We know that the set of all congruences defined on a semigroup  $S$  is a partially ordered set with respect to inclusion and relative to this partial order it forms a lattice, the lattice of congruences on  $S$ . The study of lattice of congruences on different types of semigroups such as regular semigroups and eventually regular semigroups led to breakthrough innovations made by T.E. Hall [3], LaTorre [5], S.H. Rao and P. Lakshmi [10]. The congruences that they looked into were mostly group congruences. In paper [10], S.H. Rao and P. Lakshmi characterized group congruences on eventually regular semigroups in which they used self-conjugate subsemigroups. Further studies were continued by S. Sattayaporn [11] with weakly self-conjugate subsets.

Over the years, congruence structures have been an integral part of discussion in mathematics.

In this paper, we study various types of congruences on GV-semigroups. To be more precise, we characterize least Clifford congruences on generalized quasi-orthodox GV-semigroups.

## 2. PRELIMINARIES

An element  $a$  in a semigroup  $(S, \cdot)$  is said to be regular if there exists an element  $x \in S$  such that  $axa = a$ . A semigroup  $(S, \cdot)$  is said to be regular if every element of  $S$  is regular. In this case there also exists  $y \in S$  such that  $aya = a$  and  $yay = y$ . Such an element  $y$  is called an inverse of  $a$ . An element  $a$  in a semigroup  $(S, \cdot)$  is said to be  $\pi$ -regular (or power regular) if there exists a positive integer  $n$  such that  $a^n$  is regular. Naturally, a semigroup  $(S, \cdot)$  is said to be  $\pi$ -regular (or power regular) if every element of  $S$  is  $\pi$ -regular. An element  $a$  in a semigroup  $(S, \cdot)$  is said to be completely regular if there exists an element  $x \in S$  such that  $a = axa$  and  $ax = xa$ . We know that an element  $a$  in a semigroup  $S$  is completely regular if and only if it belongs to a subgroup of  $G$ . We call a semigroup  $S$ , a completely regular semigroup if every element of  $S$  is completely regular.

An element  $a$  in a semigroup  $(S, \cdot)$  is said to be completely  $\pi$ -regular if there exists a positive integer  $n$  such that  $a^n$  is completely regular. Naturally, a semigroup  $S$  is said to be completely  $\pi$ -regular if every element of  $S$  is completely  $\pi$ -regular.

**Lemma 2.1** [7]. *Let  $S$  be a semigroup and let  $x$  be an element of  $S$  such that  $x^n$  belongs to a subgroup  $G$  of  $S$  for some positive integer  $n$ . Then, if  $e$  is the identity of  $G$ , we have*

- (a)  $ex = xe \in G$ ,
- (b)  $x^m \in G$  for any integer  $m > n$ .

Let  $a$  be a completely  $\pi$ -regular element in a semigroup  $S$ . Then  $a^n$  lies in a subgroup  $G$  of  $S$  for some positive integer  $n$ . The inverse of  $a^n$  in  $G$  is denoted by  $(a^n)^{-1}$ . From the above lemma, it follows that for a completely  $\pi$ -regular element  $a$  in a semigroup  $S$ , all its completely regular powers lie in the same subgroup of  $S$ . Let  $a^0$  be the identity of this group and  $\bar{a} = (aa^0)^{-1}$ . Then clearly,  $a^0 = a\bar{a} = \bar{a}a$  and  $aa^0 = a^0a$ .

Throughout this paper, we always let  $E(S)$  be the set of all idempotents of the semigroup  $S$ . Also we denote the set of all inverses of a regular element  $a$  in a semigroup  $S$  by  $V(a)$ . For  $a \in S$ , by “ $a^n$  is  $a$ -regular” we mean that  $n$  is the smallest positive integer for which  $a^n$  is regular.

As usual, we denote the Green's relations [4] on the semigroup  $(S, \cdot)$  by  $\mathcal{L}$ ,  $\mathcal{R}$ ,  $\mathcal{D}$ ,  $\mathcal{J}$  and  $\mathcal{H}$ . For any  $a \in S$ , we let  $H_a$  be the  $\mathcal{H}$ -class in  $S$  containing  $a$ . If  $(S, \cdot)$  be a  $\pi$ -regular semigroup, we consider the relations  $\mathcal{L}^*$ ,  $\mathcal{R}^*$ ,  $\mathcal{J}^*$ ,  $\mathcal{H}^*$  and  $\mathcal{D}^*$  defined by

$$\begin{aligned} a \mathcal{L}^* b &\text{ if and only if } a^p \mathcal{L} b^q, \\ a \mathcal{R}^* b &\text{ if and only if } a^p \mathcal{R} b^q, \\ a \mathcal{J}^* b &\text{ if and only if } a^p \mathcal{J} b^q, \\ \mathcal{H}^* &= \mathcal{L}^* \cap \mathcal{R}^* \quad \text{and} \quad \mathcal{D}^* = \mathcal{L}^* \circ \mathcal{R}^* \end{aligned}$$

where  $a^p$  is  $a$ -regular and  $b^q$  is  $b$ -regular.

A semigroup  $(S, \cdot)$  is said to be a band if each element of  $S$  is idempotent, i.e.,  $a^2 = a$  for all  $a \in S$ . A commutative band is called a semilattice. A band  $S$  is said to be a rectangular band if it satisfies the identity  $axa = a$  for all  $a, x \in S$ . A congruence  $\rho$  on a semigroup  $S$  is called a semilattice congruence if  $S/\rho$  is a semilattice. A semigroup  $S$  is called a semilattice  $Y$  of semigroups  $S_\alpha$  ( $\alpha \in Y$ ) if  $S$  admits a semilattice congruence  $\rho$  on  $S$  such that  $Y = S/\rho$  and each  $S_\alpha$  is a  $\rho$ -class mapped onto  $\alpha$  by the natural epimorphism  $\rho^\# : S \rightarrow Y$ . We write  $S = (Y; S_\alpha)$ . For other notations and terminologies not given in this paper, the reader is referred to the texts of Bogdanovic [1] and Howie [4].

### 3. COMPLETELY ARCHIMEDEAN SEMIGROUPS AND GV-SEMIGROUPS

In this section we recall some definitions and state some of their important properties.

**Definition 3.1.** A semigroup  $S$  is said to be an Archimedean semigroup if for any two elements  $a, b \in S$  there exists a positive integer  $n$  such that  $a^n \in SbS$ .

**Definition 3.2.** An Archimedean semigroup  $S$  is said to be completely Archimedean if it is completely  $\pi$ -regular.

**Definition 3.3.** Let  $I$  be an ideal of a semigroup  $S$ . We define a relation  $\rho_I$  on  $S$  by  $a\rho_I b$  if and only if either  $a, b \in I$  or  $a = b$  where  $a, b \in S$ . It is easy to verify that  $\rho_I$  is a congruence on  $S$ . This congruence is said to be Rees congruence on  $S$  and the quotient semigroup  $S/\rho_I$  contains a zero, namely  $I$ . This quotient semigroup  $S/\rho_I$  is said to be the Rees quotient semigroup and is denoted by  $S/I$ . In this case the semigroup  $S$  is said to be an ideal extension or simply an extension of  $I$  by the semigroup  $S/I$ . An ideal extension  $S$  of a semigroup  $I$  is said to be a nil-extension of  $I$  if  $S/I$  is nil semigroup, i.e., for any  $a \in S$  there exists a positive integer  $n$  such that  $a^n \in I$ .

**Theorem 3.4** [1]. *The following conditions on a semigroup are equivalent:*

- (i)  *$S$  is a completely Archimedean semigroup;*
- (ii)  *$S$  is a nil-extension of a completely simple semigroup.*

In a completely  $\pi$ -regular semigroup, regular elements may not be completely regular. A completely  $\pi$ -regular semigroup in which every regular element is completely regular is said to be a GV-semigroup. A completely  $\pi$ -regular semigroup containing a single idempotent is called a  $\pi$ -group. It is well known that a semigroup  $S$  is a  $\pi$ -group if and only if  $S$  is nil-extension of a group. The following theorem gives the complete characterization of GV-semigroups.

**Theorem 3.5** [1]. *The following conditions on a semigroup  $S$  are equivalent:*

- (i)  *$S$  is a GV-semigroup;*
- (ii)  *$S$  is  $\pi$ -regular and every  $\mathcal{H}^*$ -class of  $S$  is a  $\pi$ -group;*
- (iii)  *$S$  is semilattice of completely Archimedean semigroups.*

In this connection, it is interesting to mention that  $\mathcal{J}^*$  is a semilattice congruence on a GV-semigroup  $S$  and each  $\mathcal{J}^*$ -class is a completely Archimedean subsemigroup of  $S$ .

#### 4. GENERALIZED QUASI-ORTHODOX GV-SEMIGROUPS

In this section we define generalized quasi-orthodox semigroups and study some important properties of generalized quasi-orthodox GV-semigroups.

**Definition 4.1.** A semigroup  $(S, \cdot)$  is said to be an orthodox semigroup if  $E(S)$  forms a subsemigroup of  $S$ .

**Definition 4.2.** A semigroup  $(S, \cdot)$  is said to be a generalized quasi-orthodox semigroup if for any two elements  $e, f \in E(S)$ , there exists a positive integer  $n$  such that  $(ef)^n = (ef)^{n+1}$ .

Clearly, every orthodox semigroup is generalized quasi-orthodox. But the converse is not true in general. We cite some examples to ensure that generalized quasi-orthodox semigroup may not be an orthodox semigroup.

**Example 4.3** [6]. Let  $S = \{e, f, a, 0\}$ . On  $S$  we define a multiplication  $'\cdot'$  with the following Cayley table:

$\cdot$	$e$	$f$	$a$	$0$
$e$	$e$	$a$	$a$	$0$
$f$	$0$	$f$	$0$	$0$
$a$	$0$	$a$	$0$	$0$
$0$	$0$	$0$	$0$	$0$

Then  $(S, \cdot)$  is a generalized quasi-orthodox semigroup but not an orthodox.

**Example 4.4.** Let  $S = \{0, 1, 2, 3, 4, 5\}$ . On  $S$  we define a multiplication  $'\cdot'$  with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	2	2	3	0	0
1	3	1	3	3	1	5
2	3	2	3	3	2	0
3	3	3	3	3	3	3
4	0	1	2	3	4	5
5	5	1	1	3	5	5

Then  $(S, \cdot)$  is a semigroup with  $E(S) = \{0, 1, 3, 4, 5\}$ . Here,  $0 \cdot 1 = 2 \notin E(S)$ . Hence  $S$  is not an orthodox semigroup. But, one can easily verify that  $(S, \cdot)$  is a generalized quasi-orthodox semigroup.

**Remark 4.5.** A completely  $\pi$ -regular semigroup  $S$  is generalized quasi-orthodox if and only if for any  $e, f \in E(S)$ ,  $(ef)(ef)^0 = (ef)^0$ .

**Lemma 4.6.** Let  $S$  be a GV-semigroup;  $e, f \in E(S)$ ,  $g = (ef)^0e$  and  $h = f(ef)^0$ . Then  $ef(ef)^0 = gh$ ,  $g^2 = g$ ,  $h^2 = h$  and  $(ef) \mathcal{J}^*g \mathcal{J}^*h$ .

**Proof.** Firstly,  $gh = (ef)^0(ef)(ef)^0 = (ef)(ef)^0$ . Again,  $g^2 = (ef)^0e(ef)^0e = (ef)^0e = g$ . Similarly,  $h^2 = h$ . Since  $S$  is a semilattice of its  $\mathcal{J}^*$ -classes [1], it follows that  $(ef) \mathcal{J}^*g \mathcal{J}^*h$ . ■

**Definition 4.7.** A GV-semigroup  $S$  is said to be generalized quasi-orthodox GV-semigroup if  $S$  is a generalized quasi-orthodox semigroup.

**Theorem 4.8.** Let  $S = (Y; S_\alpha)$  be a GV-semigroup, where  $Y$  is a semilattice and  $S_\alpha$  ( $\alpha \in Y$ ) is a completely Archimedean semigroup. Then the following conditions are equivalent:

- (i)  $S$  is generalized quasi-orthodox;
- (ii) For all  $\alpha \in Y$ ,  $S_\alpha$  is orthodox;
- (iii) For all  $e \in E(S)$  and for all  $x \in S$ , there exist  $m, n \in \mathbb{N}$  such that

$$\left(x^{m-1}(x^m)^{-1}ex\right)^n = \left(x^{m-1}(x^m)^{-1}ex\right)^{n+1};$$

- (iv) For all  $a, b \in S$  there exists a positive integer  $n$  such that  $(a^0b^0)^n = (a^0b^0)^{n+1}$ .

**Proof.** (i) $\iff$ (ii) This follows from Theorem X:2.1 [1].

(i) $\Rightarrow$ (iii) For  $x \in S$  there exists a positive integer  $m$  such that  $x^m$  is  $x$ -regular. Let  $e \in E(S)$ . As  $S$  is quasi-orthodox and  $x^0 \in E(S)$ , then there exists a positive integer  $n$  such that  $(ex^0)^{n-1} = (ex^0)^n$ . Therefore,

$$\begin{aligned} \left(x^{m-1}(x^m)^{-1}ex\right)^n &= x^{m-1}(x^m)^{-1}(ex^0)^{n-1}ex \\ &= x^{m-1}(x^m)^{-1}(ex^0)^n ex \\ &= \left(x^{m-1}(x^m)^{-1}ex\right)^{n+1}. \end{aligned}$$

Hence, for all  $e \in E(S)$  and for all  $x \in S$ , there exist positive integers  $m, n \in \mathbb{N}$  such that  $(x^{m-1}(x^m)^{-1}ex)^n = (x^{m-1}(x^m)^{-1}ex)^{n+1}$ .

(iii) $\Rightarrow$ (i) Let  $e, f \in E(S)$ . Then by the given condition we have,  $(fef)^n = (fef)^{n+1}$ , for some positive integer  $n$ .

Now,  $(ef)^{n+1} = e(fef)^n = e(fef)^{n+1} = (ef)^{n+2}$ . Hence,  $S$  is generalized quasi-orthodox.

(i) $\Rightarrow$ (iv) Since  $S$  is generalized quasi-orthodox and for all  $a, b \in S$ ,  $a^0, b^0 \in E(S)$ , hence there exists a positive integer  $n$  such that  $(a^0b^0)^n = (a^0b^0)^{n+1}$ .

(iv) $\Rightarrow$ (i) This part is obvious. ■

**Lemma 4.9.** Let  $S = (Y; S_\alpha)$  be a generalized quasi-orthodox GV-semigroup, where  $Y$  is a semilattice and  $S_\alpha$  ( $\alpha \in Y$ ) is a completely Archimedean semigroup. Then for all  $\alpha \in Y$ ,  $E(S_\alpha)$  is a rectangular band and for any two elements  $a, b \in S_\alpha$ ,  $e \in E(S_\beta)$ ,  $a^0b^0 = a^0eb^0$ , where  $\alpha, \beta \in Y$  with  $\alpha \leq \beta$ .

**Proof.** Since each  $S_\alpha$  is a completely Archimedean semigroup, then  $S_\alpha$  has a completely simple kernel  $K_\alpha$ . As every element  $e \in E(S_\alpha)$  is completely regular, it follows that  $e \in E(K_\alpha)$  and thus  $E(S_\alpha) = E(K_\alpha)$ . Since  $K_\alpha$  is completely simple and orthodox, so  $E(K_\alpha)$  is a rectangular band (Corollary III.5.3 [9]). Hence  $E(S_\alpha)$  is a rectangular band.

Now  $a^0, a^0e, b^0 \in K_\alpha$ . Since  $S$  is generalized quasi-orthodox, so there exists a positive integer  $n$  such that  $(a^0e)^n = (a^0e)^{n+1}$ . Here  $(a^0e)^n \in E(K_\alpha)$ .

Therefore,

$$\begin{aligned} a^0b^0 &= a^0(a^0e)^nb^0 \\ &= a^0(a^0e)^{n+1}b^0 \\ &= a^0(a^0e)^na^0eb^0 \\ &= a^0eb^0. \end{aligned} \quad \blacksquare$$

Next we prove a very important result on generalized quasi-orthodox GV-semigroup.

**Theorem 4.10.** *Let  $S = (Y; S_\alpha)$  be a generalized quasi-orthodox GV-semigroup, where  $Y$  is a semilattice and  $S_\alpha$  ( $\alpha \in Y$ ) is a completely Archimedean semigroup. Let  $a \in S_\alpha$  be a completely regular element and  $b \in S_\beta$  where  $\alpha \leq \beta$ . Then  $ab$  is completely regular.*

**Proof.** For all  $\alpha \in Y$ ,  $S_\alpha$  is the nil-extension of its kernel  $K_\alpha$ , that is completely simple.

Clearly,  $(ab) \in S_\alpha$ . We show that  $(ab) \in K_\alpha$ . As  $S$  is a GV-semigroup, then there exists a positive integer  $n$  such that  $(ab)^n$  is completely regular and  $(ab)^n$  is in a subgroup  $G \subseteq K_\alpha$ . Let  $g$  be the identity of  $G$ . Then  $abg = gab \in G$ .

Now,

$$\begin{aligned} ab &= a^0 ab, \\ &= (a^0 ga^0) ab, \end{aligned}$$

since  $E(S_\alpha)$  is a rectangular band and  $a^0, g \in E(S_\alpha)$ .

Therefore,

$$\begin{aligned} ab &= (a^0 ga^0) ab \\ &= a^0 (ga^0 ab) \\ &= a^0 (gab) \in K_\alpha, \end{aligned}$$

since  $a^0 \in K_\alpha$ ,  $gab \in G \subseteq K_\alpha$ . Consequently,  $ab$  is completely regular. ■

## 5. LEAST CLIFFORD CONGRUENCES

In this section, we characterize least Clifford congruences on generalized quasi-orthodox GV-semigroups. For this purpose we define a relation  $\nu$  on a GV-semigroup and finally we establish a necessary and sufficient condition for  $\nu$  to be a least Clifford congruence on a GV-semigroup.

Recall that a regular semigroup in which idempotents are central is said to be a Clifford semigroup. It is interesting to mention that a semigroup  $S$  is a Clifford semigroup if and only if  $S$  is semilattice of groups.

In order to characterize further the least Clifford congruence on a GV-semigroup, we define the following relation  $\nu$ .

**Definition 5.1.** Let  $S = (Y; S_\alpha)$  be GV-semigroup, where  $Y$  is a semilattice and  $S_\alpha$  ( $\alpha \in Y$ ) is a completely Archimedean semigroup.

On  $S$  we define a relation  $\nu$  as follows. For  $a, b \in S$ ,

$$a \nu b \text{ if and only if } aa^0 = a^0 bb^0 a^0 \text{ and } bb^0 = b^0 aa^0 b^0.$$

**Theorem 5.2.** *Let  $S = (Y; S_\alpha)$  be GV-semigroup, where  $Y$  is a semilattice and  $S_\alpha$  ( $\alpha \in Y$ ) is a completely Archimedean semigroup. Then the relation  $\nu$ , as*

defined in Definition 5.1 is the least Clifford congruence on  $S$  if and only if  $S$  is generalized quasi-orthodox.

**Proof.** Let  $S = (Y; S_\alpha)$  be a generalized quasi-orthodox GV-semigroup. We prove that  $\nu$  is the least Clifford congruence on  $S$ .

We first show that  $\nu$  is an equivalence relation on  $S$ . Clearly,  $\nu$  is reflexive and symmetric.

Let  $a \nu b$  and  $b \nu c$  holds for  $a, b, c \in S$ . Then  $a, b, c \in S_\alpha$  for some  $\alpha \in Y$ .

Now,  $a \nu b$  implies  $aa^0 = a^0bb^0a^0$  and  $bb^0 = b^0aa^0b^0$ . Also,  $b \nu c$  implies  $bb^0 = b^0cc^0b^0$  and  $cc^0 = c^0bb^0c^0$ .

Therefore,

$$\begin{aligned} aa^0 &= a^0b^0cc^0b^0a^0, \\ \text{i.e., } c^0aa^0c^0 &= c^0a^0b^0cc^0b^0a^0c^0 \\ &= c^0a^0b^0cc^0, \quad [\text{since } E(S_\alpha) \text{ is a rectangular band}] \\ &= c^0a^0b^0c^0c, \\ &= c^0c, \\ &= cc^0. \end{aligned}$$

Similarly,  $aa^0 = a^0cc^0a^0$ . Thus,  $a \nu c$ . Hence  $\nu$  is an equivalence relation.

Let  $a \nu b$  and  $c \in S$ . Let  $a, b \in S_\alpha$  and  $c \in S_\beta$ . Therefore,  $aa^0 = a^0bb^0a^0$  and  $bb^0 = b^0aa^0b^0$ . By using Lemma 4.9., we have

$$\begin{aligned} (ca)^0(cb)(cb)^0(ca)^0 &= (ca)^0(cb)b^0(cb)^0(ca)^0 \\ &= (ca)^0cb^0aa^0b^0(cb)^0(ca)^0 \\ &= (ca)^0caa^0b^0(ca)^0 \\ &= (ca)^0caa^0(ca)^0 \\ &= (ca)^0ca(ca)^0 \\ &= (ca)(ca)^0. \end{aligned}$$

Similarly,  $(cb)^0(ca)(ca)^0(cb)^0 = (cb)(cb)^0$ . Therefore,  $(ca) \nu (cb)$ . Dually, we can obtain,  $(ac) \nu (bc)$ . Consequently,  $\nu$  is a congruence on  $S$ .

Now, for any  $a \in S$ ,  $e \in E(S)$  we have  $e(ae)^0 \in E(S)$ .

Also,

$$\begin{aligned} (ea)^0(ae)(ae)^0(ea)^0 &= (ea)^0(ae)(ea)^0 \\ &= (ea)^0e(ae)(ea)^0 \\ &= (ea)^0(ea)(ea)^0 \\ &= (ea)(ea)^0. \end{aligned}$$

Similarly,  $(ae)^0(ea)(ea)^0(ae)^0 = (ae)(ae)^0$ . Therefore,  $(ae) \nu (ea)$ . Thus, we conclude that  $\nu$  is a Clifford congruence on  $S$ .

Now, we verify that  $\nu$  is the least Clifford congruence on  $S$ .

Let  $\rho$  be any other Clifford congruence on  $S$  and  $a \nu b$ , for  $a, b \in S$ . Then,  $(aa^0) \nu (bb^0)$ . Now,  $aa^0 = a^0 bb^0 a^0 = (a^0 b^0 aa^0 b^0 a^0) \rho (b^0 a^0 aa^0 a^0 b^0) = b^0 aa^0 b^0 = bb^0$ . Hence,  $(aa^0) \rho (bb^0)$ . Now,  $a \rho (aa^0) \rho (bb^0) \rho b$  implies,  $\nu \subseteq \rho$ . Hence,  $\nu$  is the least Clifford congruence on  $S$ .

To prove the converse, let  $\nu$  be the least Clifford congruence on a GV-semigroup  $S$ . We show that  $S$  is generalized quasi-orthodox. Now, let  $e, f \in E(S)$ . Then, by definition,  $(ef) \nu (fe)$ . This implies,  $(ef) \nu = (fe) \nu$ . Let  $(ef)^n$  be  $(ef)$ -regular. Then,  $(ef)^{n+1} \nu = (fe) \nu (ef)^n \nu$ , i.e.,  $(ef)^{n+1} \nu = (ef)^n \nu$ . This implies  $((ef)(ef)^0) \nu = (ef)^0 \nu$ , i.e.,  $(ef)(ef)^0 = (ef)^0$ , i.e.,  $(ef)^{n+1} = (ef)^n$ . This proves that  $S$  is generalized quasi-orthodox. ■

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