Discussiones Mathematicae
General Algebra and Applications 20 (2000) 77–86

ON FUZZY TOPOLOGICAL BCC-ALGEBRAS[1]

Wiesław A. Dudek
Institute of Mathematics Technical University
Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland
e-mail: dudek @ im.pwr.wroc.pl

Young Bae Jun and Sung Min Hong
Department of Mathematics Education
Gyeongsang National University
Chinju 660–701, Korea
e-mail: ybjun @ nongae.gsnu.ac.kr

Abstract

We describe properties of subalgebras and BCC-ideals in BCC-algebras with a topology induced by a family of fuzzy sets.

Keywords: BCC-algebra, fuzzy subalgebra, fuzzy topological subalgebra.

1991 Mathematics Subject Classification: 06F35, 03G25, 94D05.

1. Introduction

In 1966, Y. Imai and K. Iséki (cf. [6]) defined a class of algebras of type (2,0) called *BCK-algebras* which generalize the notion of algebra of sets with the set subtraction as the only fundamental non-nullary operation, on the other hand the notion of implication algebra (cf. [7]). The class of all BCK-algebras is a quasivariety. K. Iséki posed an interesting problem whether the class of BCK-algebras is a variety. That problem was solved by A. Wroński [11] who proved that BCK-algebras do not form a variety. In

[1]The second and third authors were supported by the Basic Science Research Institute Program, Ministry of Education, 1997, Project No. BSRI-97-1406.

connection with this problem, Y. Komori [8] introduced the notion of BCC-algebras, and W.A. Dudek (cf. [1], [2]) redefined the notion of BCC-algebras by using a dual form of the ordinary definition in the sense of Y. Komori. In [4], W.A. Dudek and X.H. Zhang introduced a new notion of ideals in BCC-algebras and described connections between such ideals and congruences. W.A. Dudek and Y.B. Jun (cf. [3]) considered the fuzzification of ideals in BCC-algebras. They proved that every fuzzy BCC-ideal of a BCC-algebra is a fuzzy BCK-ideal, and showed that the converse is not true by providing a counterexample. They also proved that in a BCC-algebra every fuzzy BCK-ideal is a fuzzy BCC-subalgebra; and that in a BCK-algebra the notion of a fuzzy BCK-ideal and a fuzzy BCC-ideal coincide. The concept of a fuzzy set, which was introduced in [12], provides a natural framework for generalizing many of the concepts of general topology to what might be called fuzzy topological spaces. D.H. Foster [5] combined the structure of a fuzzy topological spaces with that of a fuzzy group, introduced by A. Rosenfeld [10], to formulate the elements of a theory of fuzzy topological groups. In the present paper, we introduce the concept of fuzzy topological subalgebras of BCC-algebras and apply some of Foster's results on homomorphic images and inverse images to fuzzy topological subalgebras.

2. Preliminaries

In the present paper we will use the definition of BCC-algebras in the sense of [2] and [4].

A nonempty set X with a constant 0 and a binary operation denoted by juxtaposition is called a *BCC-algebra* if for all $x, y, z \in X$ the following axioms hold:

(I) $((xy)(zy))(xz) = 0$,

(II) $xx = 0$,

(III) $0x = 0$,

(IV) $x0 = x$,

(V) $xy = 0$ *and* $yx = 0$ *imply* $x = y$.

In a BCC-algebra, the following holds:

(1) $$(xy)x = 0.$$

Any BCK-algebra is a BCC-algebra, but there are BCC-algebras which are not BCK-algebras (cf. [2]). Note that a BCC-algebra is a BCK-algebra iff it satisfies:

(2) $$(xy)z = (xz)y.$$

A nonempty subset S of a BCC-algebra X is called *a subalgebra* of X if it is closed under the BCC-operation. Such subalgebra contains obviously the constant 0 and is a BCC-algebra, but some subalgebras may be also BCK-algebras. Moreover, there are BCC-algebras in which all subalgebras are BCK-algebras (cf. [1]).

We now review some fuzzy logic concepts. Let X be a set. A *fuzzy set* A in X is characterized by a membership function $\mu_A : X \to [0,1]$. A fuzzy set is *empty* iff its membership function is identically zero on X. If A and B are two fuzzy sets on X with respective membership functions μ_A and μ_B, then

$$A \subseteq B \iff (\forall\, x \in X)\ [\mu_A(x) \le \mu_B(x)]$$
$$A = B \iff (\forall\, x \in X)\ [\mu_A(x) = \mu_B(x)].$$

In the case $A \subset B$ we say that a fuzzy set A is *smaller* than B (cf. [12]).

The *union* of two fuzzy sets A and B is a fuzzy set C, written as $C = A \cup B$, whose membership function is related to those A and B by

$$(\forall\, x \in X)\ [\mu_C(x) = \max\{\mu_A(x)\,,\, \mu_B(x)\}]\ .$$

The union of A and B is the smallest fuzzy set containing both A and B.

The *intersection* of two fuzzy sets A and B is a fuzzy set D, written as $D = A \cap B$, whose membership function is related to those of A and B by

$$(\forall\, x \in X)\ [\mu_D(x) = \min\{\mu_A(x)\,,\, \mu_B(x)\}]\ .$$

The intersection of A and B is the largest fuzzy set which is contained in both A and B.

As in the case of ordinary sets, A and B are *disjoint* if $A \cap B$ is empty. Note that fuzzy sets in X constitute a distributive lattice with 0 and 1.

Let α be a mapping from the set X to a set Y. Let B be a fuzzy set in Y with membership function μ_B. The *inverse image* of B, denoted $\alpha^{-1}(B)$, is the fuzzy set in X with membership function $\mu_{\alpha^{-1}(B)}(x) = \mu_B(\alpha(x))$ for all $x \in X$. Conversely, let A be a fuzzy set in X with membership function μ_A. Then the *image* of A, denoted $\alpha(A)$, is the fuzzy set in Y such that

$$\mu_{\alpha(A)}(y) = \begin{cases} \sup\limits_{z \in \alpha^{-1}(y)} \mu_A(z), & \text{if } \alpha^{-1}(y) = \{x : \alpha(x) = y\} \text{ is nonempty,} \\ 0, & \text{otherwise.} \end{cases}$$

A *fuzzy topology* on a set X is a family $\mathcal{T}$ of fuzzy subsets in X which satisfies the following conditions:

(i) For all $c \in [0,1]$, $k_c \in \mathcal{T}$, where k_c have constant membership functions with the value c,

(ii) If $A, B \in \mathcal{T}$, then $A \cap B \in \mathcal{T}$,

(iii) If $A_j \in \mathcal{T}$ for all $j \in J$, then $\bigcup\limits_{j \in J} A_j \in \mathcal{T}$.

The pair $(X, \mathcal{T})$ is called a *fuzzy topological space* and members of $\mathcal{T}$ are *open fuzzy subsets.*

Let A be a fuzzy subset in X and $\mathcal{T}$ a fuzzy topology on X. Then the *induced fuzzy topology* on A is the family of fuzzy subsets of A which are the intersection with A of $\mathcal{T}$-open fuzzy subsets in X. The induced fuzzy topology is denoted by $\mathcal{T}_A$, and the pair $(A, \mathcal{T}_A)$ is called a *fuzzy subspace* of $(X, \mathcal{T})$.

Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be two fuzzy topological spaces. A mapping α of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$ is *fuzzy continuous* if for each open fuzzy set U in $\mathcal{U}$, the inverse image $\alpha^{-1}(U)$ is in $\mathcal{T}$. Conversely, α is *fuzzy open* if for each open fuzzy set V in $\mathcal{T}$, the image $\alpha(V)$ is in $\mathcal{U}$.

Let $(A, \mathcal{T}_A)$ and $(B, \mathcal{U}_B)$ be fuzzy subspaces of fuzzy topological spaces $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ respectively, and let α be a mapping $(X, \mathcal{T}) \to (Y, \mathcal{U})$. Then α is a mapping of $(A, \mathcal{T}_A)$ into $(B, \mathcal{U}_B)$ if $\alpha(A) \subset B$. Furthermore α is *relatively fuzzy continuous* if for each open fuzzy set V' in $\mathcal{U}_B$, the intersection $\alpha^{-1}(V') \cap A$ is in $\mathcal{T}_A$. Conversely, α is *relatively fuzzy open* if for each open fuzzy set U' in $\mathcal{T}_A$, the image $\alpha(U')$ is in $\mathcal{U}_B$.

Lemma 2.1 [5]**.** *Let $(A, \mathcal{T}_A)$, $(B, \mathcal{U}_B)$ be fuzzy subspaces of fuzzy topological spaces $(X, \mathcal{T})$, $(Y, \mathcal{U})$ respectively, and let α be a fuzzy continuous mapping of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$ such that $\alpha(A) \subset B$. Then α is a relatively fuzzy continuous mapping of $(A, \mathcal{T}_A)$ into $(B, \mathcal{U}_B)$.*

3. Fuzzy topological subalgebras

Definition 3.1 [3]**.** A fuzzy subset F in a BCC-algebra X with membership function μ_F is called a *fuzzy subalgebra* of X if

$$(\forall x, y \in X) \;\; [\mu_F(xy) \geq \min\{\mu_F(x),\, \mu_F(y)\}].$$

Example 3.2 [3]**.** Let $X = \{0, a, b, c, d\}$ be a set with Cayley table of the binary operation as follows:

$\cdot$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	0	0
c	c	c	a	0	0
d	d	c	d	c	0

Table 1

Then X is a BCC-algebra ([4]) which is not a BCK-algebra since $(da)b \neq (db)a$. By routine calculations we know that a fuzzy subset F in X with membership function μ_F defined by $\mu_F(d) = 0.4$ and $\mu_F(x) = 0.8$ for all $x \neq d$ is a fuzzy subalgebra of X. ■

Proposition 3.3. *Let α be a homomorphism of a BCC-algebra X into a BCC-algebra Y and G a fuzzy subalgebra of Y with membership function μ_G. Then the inverse image $\alpha^{-1}(G)$ of G is a fuzzy subalgebra of X.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned} \mu_{\alpha^{-1}(G)}(xy) &= \mu_G(\alpha(xy)) = \mu_G(\alpha(x)\alpha(y)) \\ &\geq \min\{\mu_G(\alpha(x)),\, \mu_G(\alpha(y))\} \\ &= \min\{\mu_{\alpha^{-1}(G)}(x),\, \mu_{\alpha^{-1}(G)}(y)\}. \end{aligned}$$

This completes the proof. ■

For images, we need the following definition [10].

Definition 3.4. A fuzzy subset F in a BCC-algebra X with membership function μ_F is said to have the *sup property* if, for any subset $T \subset X$, there exists $t_0 \in T$ such that

$$\mu_F(t_0) = \sup_{t \in T} \mu_F(t)\,.$$

Proposition 3.5. *Let α be a homomorphism of a BCC-algebra X onto a BCC-algebra Y and let F be a fuzzy subalgebra of X that has the sup property. Then the image $\alpha(F)$ of F is a fuzzy subalgebra of Y.*

Proof. For $u, v \in Y$, let $x_0 \in \alpha^{-1}(u)$, $y_0 \in \alpha^{-1}(v)$ such that

$$\mu_F(x_0) = \sup_{t \in \alpha^{-1}(u)} \mu_F(t), \qquad \mu_F(y_0) = \sup_{t \in \alpha^{-1}(v)} \mu_F(t) .$$

Then, by the definition of $\mu_{\alpha(F)}$, we have

$$\begin{aligned} \mu_{\alpha(F)}(uv) &= \sup_{t \in \alpha^{-1}(uv)} \mu_F(t) \geq \mu_F(x_0 y_0) \\ &\geq \min\{\mu_F(x_0),\ \mu_F(y_0)\} \\ &= \min\{ \sup_{t \in \alpha^{-1}(u)} \mu_F(t),\ \sup_{t \in \alpha^{-1}(v)} \mu_F(t)\} \\ &= \min\{\mu_{\alpha(F)}(u),\ \mu_{\alpha(F)}(v)\} , \end{aligned}$$

ending the proof. ■

For any BCC-algebra X and any element $a \in X$ we use a_r to denote the selfmap of X defined by $a_r(x) = xa$ for all $x \in X$.

Definition 3.6. Let X be a BCC-algebra and $\mathcal{T}$ a fuzzy topology on X. Let F be a fuzzy subalgebra of X with induced topology $\mathcal{T}_F$. Then F is called a *fuzzy topological subalgebra* of X if for each $a \in X$ the mapping $a_r : x \mapsto xa$ of $(F, \mathcal{T}_F) \to (F, \mathcal{T}_F)$ is relatively fuzzy continuous.

Theorem 3.7. *Given BCC-algebras X, Y and a homomorphism $\alpha : X \to Y$, let $\mathcal{U}$ and $\mathcal{T}$ be fuzzy topologies on Y and X respectively, such that $\mathcal{T} = \alpha^{-1}(\mathcal{U})$. Let G be a fuzzy topological subalgebra of Y with membership function μ_G. Then $\alpha^{-1}(G)$ is a fuzzy topological subalgebra of X with membership function $\mu_{\alpha^{-1}(G)}$.*

Proof. We have to show that, for each $a \in X$, the mapping

$$a_r : x \mapsto xa \quad \text{ of } \quad (\alpha^{-1}(\mathrm{G}),\, \mathcal{T}_{\alpha^{-1}(\mathrm{G})}) \to (\alpha^{-1}(\mathrm{G}),\, \mathcal{T}_{\alpha^{-1}(\mathrm{G})})$$

is relatively fuzzy continuous. Let U be an open fuzzy set in $\mathcal{T}_{\alpha^{-1}(G)}$ on $\alpha^{-1}(G)$. Since α is a fuzzy continuous mapping of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$, it follows from Lemma 2.1 that α is a relatively fuzzy continuous mapping of $(\alpha^{-1}(G), \mathcal{T}_{\alpha^{-1}(G)})$ into $(G, \mathcal{U}_G)$. Note that there exists an open fuzzy set $V \in \mathcal{U}_G$ such that $\alpha^{-1}(V) = U$. The membership function of $a_r^{-1}(U)$ is given by

$$\mu_{a_r^{-1}(U)}(x) = \mu_U(a_r(x)) = \mu_U(xa) = \mu_{\alpha^{-1}(V)}(xa) = \mu_V(\alpha(xa)) = \mu_V(\alpha(x)\alpha(a)).$$

As G is a fuzzy topological subalgebra of Y, the mapping

$$b_r : y \mapsto yb \quad \text{of} \quad (G,\mathcal{U}_G) \to (G,\mathcal{U}_G)$$

is relatively fuzzy continuous for each $b \in Y$. Hence

$$\begin{aligned} \mu_{a_r^{-1}(U)}(x) &= \mu_V(\alpha(x)\alpha(a)) = \mu_V(\alpha(a)_r(\alpha(x))) \\ &= \mu_{\alpha(a)_r^{-1}(V)}(\alpha(x)) = \mu_{\alpha^{-1}(\alpha(a)_r^{-1}(V))}(x)\,, \end{aligned}$$

which implies that $a_r^{-1}(U) = \alpha^{-1}(\alpha(a)_r^{-1}(V))$ so that

$$a_r^{-1}(U) \cap \alpha^{-1}(G) = \alpha^{-1}(\alpha(a)_r^{-1}(V)) \cap \alpha^{-1}(G)$$

is open in the induced fuzzy topology on $\alpha^{-1}(G)$. This completes the proof. ■

We say that the membership function μ_G of a fuzzy subalgebra G of a BCC-algebra X is *α-invariant* [10] if, for all $x, y \in X$, $\alpha(x) = \alpha(y)$ implies $\mu_G(x) = \mu_G(y)$.

Clearly, a homomorphic image $\alpha(G)$ of G is then a fuzzy subalgebra.

Theorem 3.8. *Given BCC-algebras X, Y and a homomorphism α of X onto Y, let $\mathcal{T}$ be a fuzzy topology on X and $\mathcal{U}$ be the fuzzy topology on Y such that $\alpha(\mathcal{T}) = \mathcal{U}$. Let F be a fuzzy topological subalgebra of X. If the membership function μ_F of F is α-invariant, then $\alpha(F)$ is a fuzzy topological subalgebra of Y.*

Proof. It is sufficient to show that the mapping

$$b_r : y \mapsto yb \quad \text{of} \quad (\alpha(F), \mathcal{U}_{\alpha(F)}) \to (\alpha(F), \mathcal{U}_{\alpha(F)})$$

is relatively fuzzy continuous for each $b \in Y$. Note that α is relatively fuzzy open; for if $U' \in \mathcal{T}_F$, there exists $U \in \mathcal{T}$ such that $U' = U \cap F$ and by the α-invariance of μ_F,

$$\alpha(U') = \alpha(U) \cap \alpha(F) \in \mathcal{U}_{\alpha(F)}\,.$$

Let V' be an open fuzzy set in $\mathcal{U}_{\alpha(F)}$. Since α is onto, for each $b \in Y$ there exists $a \in X$ such that $b = \alpha(a)$. Hence

$$\begin{aligned} \mu_{\alpha^{-1}(b_r^{-1}(V'))}(x) &= \mu_{\alpha^{-1}(\alpha(a)_r^{-1}(V'))}(x) = \mu_{\alpha(a)_r^{-1}(V')}(\alpha(x)) \\ &= \mu_{V'}(\alpha(a)_r(\alpha(x))) = \mu_{V'}(\alpha(x)\alpha(a)) \\ &= \mu_{V'}(\alpha(xa)) = \mu_{\alpha^{-1}(V')}(xa) \\ &= \mu_{\alpha^{-1}(V')}(a_r(x)) = \mu_{a_r^{-1}(\alpha^{-1}(V'))}(x)\,, \end{aligned}$$

which implies that $\alpha^{-1}(b_r^{-1}(V')) = a_r^{-1}(\alpha^{-1}(V'))$.

By hypothesis, $a_r : x \mapsto xa$ is a relatively fuzzy continuous mapping: $(F, \mathcal{T}_F) \to (F, \mathcal{T}_F)$ and α is a relatively fuzzy continuous mapping: $(F, \mathcal{T}_F) \to (\alpha(F), \mathcal{U}_{\alpha(F)})$. Hence

$$\alpha^{-1}(b_r^{-1}(V')) \cap G = a_r^{-1}(\alpha^{-1}(V')) \cap F$$

is open in $\mathcal{T}_F$. Since α is relatively fuzzy open,

$$\alpha(\alpha^{-1}(b_r^{-1}(V')) \cap F) = b_r^{-1}(V') \cap \alpha(F)$$

is open in $\mathcal{U}_{\alpha(F)}$. This completes the proof. ■

4. Fuzzy topological ideals

First we briefly review the concepts of fuzzy ideals of BCC-algebras (cf. [3]).

Definition 4.1. A fuzzy subset A in X with membership function μ_A is called a *fuzzy BCK-ideal* of X if

(a) $(\forall x \in X)\ [\mu_A(0) \geq \mu_A(x)]$,
(b) $(\forall x, y \in X)\ [\mu_A(x) \geq \min\{\mu_A(xy),\ (\mu_A(y)\}]$.

Definition 4.2. A fuzzy subset A in X with membership function μ_A is called a *fuzzy BCC-ideal* of X if

(a) $(\forall x \in X)\ [\mu_A(0) \geq \mu_A(x)]$,
(c) $(\forall x, y, z \in X)\ [\mu_A(xz) \geq \min\{\mu_A((xy)z),\ \mu_A(y)\}]$.

Putting $z = 0$ in (c) we see that a fuzzy BCC-ideal is a fuzzy BCK-ideal. The converse is not true, in general (cf. [3]).

Proposition 4.3. *Let α be a homomorphism of a BCC-algebra X into a BCC-algebra Y and B a fuzzy BCC-ideal of Y with membership function μ_B. Then the inverse image $\alpha^{-1}(B)$ of B is a fuzzy BCC-ideal of X.*

Proof. Since α is a homomorphism of $(X, \cdot, 0)$ into $(Y, \cdot, \theta)$, then $\alpha(0) = \theta$ and, by the assumption, $\mu_B(\alpha(0)) = \mu_B(\theta) \geq \mu_B(y)$ for every $y \in Y$. In particular, $\mu_B(\alpha(0)) \geq \mu_B(\alpha(x))$ for $x \in X$. Thus $\mu_{\alpha^{-1}(B)}(0) \geq \mu_{\alpha^{-1}(B)}(x)$, which proves (a).

Now, let $x, y, z \in X$. Then, by the assumption on μ_B, we have

$$\begin{aligned} \mu_{\alpha^{-1}(B)}(xz) &= \mu_B(\alpha(xz)) = \mu_B(\alpha(x)\alpha(z)) \\ &\geq \min\{\mu_B((\alpha(x)u)\alpha(z)),\ \mu_B(u)\} \end{aligned}$$

for all $\alpha(x), u, \alpha(z) \in Y$. In particular, for $u = \alpha(y)$, this gives

$$\begin{aligned} \mu_{\alpha^{-1}(B)}(xz) &\geq \min\{\mu_B((\alpha(x)\alpha(y))\alpha(z)),\ \mu_B(\alpha(y))\} \\ &= \min\{\mu_B(\alpha((xy)z)),\ \mu_B(\alpha(y))\} \\ &= \min\{\mu_{\alpha^{-1}(B)}((xy)z),\ \mu_{\alpha^{-1}(B)}(y)\}\,, \end{aligned}$$

which proves (c). The proof is complete. ∎

Putting in the above proof $z = 0$, we obtain

Corollary 4.4. *Let α be a homomorphism of a BCC-algebra X into a BCC-algebra Y and B a fuzzy BCK-ideal of Y with membership function μ_B. Then the inverse image $\alpha^{-1}(B)$ of B is a fuzzy BCK-ideal of X.* ∎

Since any fuzzy BCC-ideal (BCK-ideal) is a fuzzy subalgebra (cf. [3]), then a fuzzy topological BCC-ideal (BCK-ideal) is a fuzzy topological subalgebra and as a consequence of the above results and Theorem 3.7, we obtain

Corollary 4.5. *Given BCC-algebras X, Y and a homomorphism $\alpha\colon X \to Y$, let $\mathcal{U}$ and $\mathcal{T}$ be fuzzy topologies on Y and X respectively, such that $\mathcal{T} = \alpha^{-1}(\mathcal{U})$. Let G be a fuzzy topological BCC-ideal (BCK-ideal) of Y with membership function μ_G. Then $\alpha^{-1}(G)$ is a fuzzy topological BCC-ideal (BCK- ideal) of X with membership function $\mu_{\alpha^{-1}(G)}$.* ∎

It is not difficult to see that if the membership function μ_G of a fuzzy BCC-ideal (BCK-ideal) G of a BCC-algebra X is α-invariant, then a homomorphic image $\alpha(G)$ of G is a fuzzy BCC-ideal (BCK-ideal). Thus from Theorem 3.8 it follows

Corollary 4.6. *Given BCC-algebras X, Y and a homomorphism α of X onto Y, let $\mathcal{T}$ be a fuzzy topology on X and $\mathcal{U}$ be the fuzzy topology on Y such that $\alpha(\mathcal{T}) = \mathcal{U}$. Let F be a fuzzy topological BCC-ideal (BCK-ideal) of X. If the membership function μ_F of F is α-invariant, then $\alpha(F)$ is a fuzzy topological BCC-ideal (BCK-ideal) of Y.* ∎

References

[1] W.A. Dudek, *The number of subalgebras of finite BCC-algebras*, Bull. Inst. Math. Acad. Sinica **20** (1992), 129–136.

[2] W.A. Dudek, *On proper BCC-algebras*, Bull. Inst. Math. Acad. Sinica **20** (1992), 137-150.

[3] W.A. Dudek and Y.B. Jun, *Fuzzy BCC-ideals in BCC-algebras*, Math. Montisnigri **10** (1999), 21–30.

[4] W.A. Dudek and X.H. Zhang, *On ideals and congruences in BCC-algebras*, Czechoslovak Math. J. **48** (**123**) (1998), 21– 29.

[5] D.H. Foster, *Fuzzy topological groups*, J. Math. Anal. Appl. **67** (1979), 549–564.

[6] Y. Imai and K. Iséki, *On axiom system of propositional calculi XIV*, Proc. Japan Acad. **42** (1966), 19–22.

[7] K. Iséki and S. Tanaka, *An introduction to the theory of BCK-algebras*, Math. Japon. **23** (1978), 1–26.

[8] Y. Komori, *The class of BCC-algebras is not a variety*, Math. Japon. **29** (1984), 391–394.

[9] J. Meng and Y. B. Jun, *BCK-algebras*, Kyungmoonsa, Seoul, Korea 1994.

[10] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl. **35** (1971), 512–517.

[11] A. Wroński, *BCK-algebras do not form a variety*, Math. Japon. **28** (1983), 211–213.

[12] L.A. Zadeh, *Fuzzy sets*, Inform. Control **8** (1965), 338–353.

Received 3 September 1998
Revised 26 March 1999