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ORDER OF FINITE SOFT QUASIGROUPS WITH APPLICATION TO EGALITARIANISM

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Abstract

In this work, a soft set (F, A) was introduced over a quasigroup $(Q, \cdot)$ and the study of finite soft quasigroup was carried out, motivated by the study of algebraic structures of soft sets. By introducing the order of a finite soft quasigroup, various inequality relationships that exist between the order of a finite quasigroup, the order of its soft quasigroup and the cardinality of its set of parameters were established. By introducing the arithmetic mean $\mathcal{AM}(F, A)$ and geometric mean $\mathcal{GM}(F, A)$ of a finite soft quasigroup (F, A), a sort of Lagrange's Formula $|(F, A)| = |A|\mathcal{AM}(F, A)$ for finite soft quasigroup was gotten. Some of the inequalities gotten gave an upper bound for the order of a finite soft quasigroup in terms of the order of its quasigroup and cardinality of its set of parameters, and a lower bound for the order of the quasigroup in terms of the arithmetic mean of the finite soft quasigroup. A chain of inequalities called the Maclaurin's inequality for

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any finite soft quasigroup $(F, A)_{(Q,\cdot)}$ was shown to exist. A necessary and sufficient condition for a type of finite soft quasigroup to be extensible to a finite super soft quasigroup was established. This result is of practical use whenever a larger set of parameters is required. The results therein were illustrated with examples. Application to uniformity, equality and equity in distribution for social living is considered.

Keywords: soft sets, quasigroups, soft quasigroups, soft subquasigroups, arithmetic and geometric mean, inequalities.

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