# REVISITING THE REPRESENTATION THEOREM OF FINITE DISTRIBUTIVE LATTICES WITH PRINCIPAL CONGRUENCES. A PROOF-BY-PICTURE APPROACH 

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#### Abstract

A classical result of R.P. Dilworth states that every finite distributive lattice $D$ can be represented as the congruence lattice of a finite lattice $L$. A sharper form was published in G. Grätzer and E.T. Schmidt in 1962, adding the requirement that all congruences in $L$ be principal. Another variant, published in 1998 by the authors and E.T. Schmidt, constructs a planar semimodular lattice $L$. In this paper, we merge these two results: we construct $L$ as a planar semimodular lattice in which all congruences are principal. This paper relies on the techniques developed by the authors and E.T. Schmidt in the 1998 paper.


Keywords: principal congruence, finite distributive lattice.
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