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GENERALIZED ROUGH SETS APPLIED TO *BCK/BCI*-ALGEBRAS

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Abstract

The concept of a (strong) set-valued BCK/BCI-morphism in BCK/BCI-algebras is considered, and several properties are investigated. Conditions for a set-valued mapping to be a set-valued BCK/BCI-morphism are given. Using the concept of generalized approximation space, generalized rough subalgebra (ideal) in BCK/BCI-algebras are introduced, and investigate their properties. Using the concept of generalized approximation space and ideal of BCK/bCI-algebra, another type of generalized lower and upper approximations based on the ideal is considered, and then several properties are investigated.

Keywords: generalized approximation space, generalized rough set, (strong) set-valued BCK/BCI-morphism, generalized lower rough subalgebra (ideal), generalized upper rough subalgebra (ideal).

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