Discussiones Mathematicae
General Algebra and Applications 41 (2021) 153–155
doi:10.7151/dmgaa.1359

Full PDF

DM-GAA Page

APPLYING THE CZÉDLI-SCHMIDT SEQUENCES TO CONGRUENCE PROPERTIES OF PLANAR SEMIMODULAR LATTICES

G. Grätzer

Department of Mathematics
University of Manitoba
Winnipeg, MB R3T 2N2, Canada

e-mail: gratzer@me.com

Abstract

Following Grätzer and Knapp, 2009, a planar semimodular lattice L is *rectangular*, if the left boundary chain has exactly one doubly-irreducible element, c_l, and the right boundary chain has exactly one doubly-irreducible element, c_r, and these elements are complementary.

The Czédli-Schmidt Sequences, introduced in 2012, construct rectangular lattices. We use them to prove some structure theorems. In particular, we prove that for a slim (no M_3 sublattice) rectangular lattice L, the congruence lattice $\operatorname{Con} L$ has exactly $\operatorname{length}[c_l, 1] + \operatorname{length}[c_r, 1]$ dual atoms and a dual atom in $\operatorname{Con} L$ is a congruence with exactly two classes. We also describe the prime ideals in a slim rectangular lattice.

Keywords: lattice, congruence, semimodular, planar, slim.

2010 Mathematics Subject Classification: Primary: 06C10, Secondary: 06B10.

References

[1] K. Adaricheva, V.A. Gorbunov and V.I. Tumanov, *Join-semidistributive lattices and convex geometries*, Adv. Math. **173** (2003) 1–49.
doi:10.1016/s0001-8708(02)00011-7

[2] S.P. Avann, *Locally atomic upper locally distributive lattices*, Math. Ann. **175** (1968) 320–336.
doi:10.1007/bf02063217

[3] G. Birkhoff, Lattice Theory, Third Edition, American Mathematical Society Colloquium Publications, Vol. XXV (American Mathematical Society, Providence, R.I., 1967).

[4] G. Czédli, *A Horn sentence in coalition lattices*, Acta Math. Hungarica **72** (1996) 99–104.
doi:10.1007/bf00053700

[5] G. Czédli, *Sums of lattices and a relational category*, Order **26** (2009) 309–318.
doi:10.1007/s11083-009-9127-7

[6] G. Czédli, *A note on congruence lattices of slim semimodular lattices*, Algebra Univ. **72** (2014) 225–230.
doi:10.1007/s00012-014-0286-z

[7] G. Czédli, *Finite convex geometries of circles*, Discrete Math. **330** (2014) 61–75.
doi:10.1016/j.disc.2014.04.017

[8] G. Czédli and G. Grätzer, Planar Semimodular Lattices: Structure and Diagrams, Chapter 3 in [25].
doi:10.1007/978-3-319-06413-0_3

[9] G. Czédli, L. Ozsvárt and B. Udvari, *How many ways can two composition series intersect*?, Discrete Math. **312** (2012) 3523–3536.
doi:10.1016/j.disc.2012.08.003

[10] G. Czédli and E.T. Schmidt, *Slim semimodular lattices* I. *A visual approach*, Order **29** (2012) 481–497.
doi:10.1007/s11083-011-9215-3

[11] R.P. Dilworth, *Lattices with unique irreducible decompositions*, Ann. of Math. **41** (1940) 771–777.
doi:10.1007/978-1-4899-3558-8_10

[12] G. Grätzer, Lattice Theory: Foundation (Birkhäuser Verlag, Basel, 2011).
doi:10.1007/978-3-0348-0018-1

[13] G. Grätzer, Planar Semimimodular Lattices: Congruences, Chapter 4 in [24].
doi:10.1007/978-3-319-06413-0_4

[14] G. Grätzer, The Congruences of a Finite Lattice, A *Proof-by-Picture* Approach, Second Edition (Birkhäuser, 2016).
doi:10.1007/978-3-319-38798-7

[15] G. Grätzer, *Congruences in slim, planar, semimodular lattices: The Swing Lemma*, Acta Sci. Math. (Szeged) **81** (2015) 381–397.
doi:10.14232/actasm-015-757-1

[16] G. Grätzer, *Congruences of fork extensions of lattices*, Algebra Univ. **76** (2016) 139–154.
doi:10.1007/s00012-016-0394-z

[17] G. Grätzer, *Notes on planar semimodular lattices*, VIII, *Congruence lattices of SPS lattices*, Algebra Univ.
doi:10.1007/s00012-020-0641-1

[18] G. Grätzer and E. Knapp, *Notes on planar semimodular lattices*, I, *Construction*, Acta Sci. Math. (Szeged) **73** (2007) 445–462. Acta Sci. Math. (Szeged) **74** (2008) 37–47.

[19] G. Grätzer and E. Knapp, *A note on planar semimodular lattices*, Algebra Univ. **58** (2008) 497–499.
doi:10.1007/s00012-008-2089-6

[20] G. Grätzer and E. Knapp, *Notes on planar semimodular lattices*, II, *Congruences*, Acta Sci. Math. (Szeged) **74** (2008) 37–47.

[21] G. Grätzer and E. Knapp, *Notes on planar semimodular lattices*, III, *Rectangular lattices*, Acta Sci. Math. (Szeged) **75** (2009) 29–48.

[22] G. Grätzer and E. Knapp, *Notes on planar semimodular lattices*, IV, *The size of a minimal congruence lattice representation with rectangular lattices*, Acta Sci. Math. (Szeged) **76** (2010) 3–26.

[23] G. Grätzer, H. Lakser, and E.T. Schmidt, *Congruence lattices of small planar lattices*, Proc. Amer. Math. Soc. **123** (1995) 2619–2623.
doi:10.2307/2160551

[24] G. Grätzer and F. Wehrung eds., Lattice Theory: Special Topics and Applications, Volume 1 (Birkhäuser Verlag, Basel, 2014).
doi:10.1007/978-3-319-06413-0

[25] G. Grätzer and F. Wehrung eds., Lattice Theory: Special Topics and Applications, Volume 2 (Birkhäuser Verlag, Basel, 2016).
doi:10.1007/978-3-319-44236-5

[26] D. Kelly and I. Rival, *Planar lattices*, Canad. J. Math. **27** (1975) 636–665.
doi:10.4153/cjm-1975-074-0

[27] S. MacLane, *A conjecture of Ore on chains in partially ordered sets*, Bull. Amer. Math. Soc. **49** (1943) 567–568.
doi:10.1090/s0002-9904-1943-07972-4

[28] O. Ore, *Chains in partially ordered sets*, Bull. Amer. Math. Soc. **49** (1943) 558–566.
doi:10.1090/s0002-9904-1943-07970-0

[29] P.M. Whitman, *Free lattices*, Ann. of Math. **42** (2) (1941) 325–330.
doi:10.2307/1969001

[30] P.M. Whitman, *Free lattices* II, Ann. of Math. **43** (2) (1942) 104–115.
doi:10.2307/1968883

Received 24 March 2020
Revised 4 December 2020
Accepted 6 December 2020