

## UNITARY INVERTIBLE GRAPHS OF FINITE RINGS

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### Abstract

Let  $R$  be a finite commutative ring with unity. In this paper, we consider set of additive and mutual additive inverses of group units of  $R$  and obtain interrelations between them. In general  $\varphi(Z_n)$  is even, however we demonstrate that  $\varphi(R)$  is odd for any finite commutative ring with unity of  $\text{Char}(R) \neq 2$ . Further, we present unitary invertible graph related with self and mutual additive inverses of group units. At long last, we establish a formula for counting the total number of basic and non-basic triangles in the unitary invertible graph.

**Keywords:** finite commutative rings, additive and mutual additive inverses, Euler-function, unitary invertible graphs, basic and non-basic triangles.

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### REFERENCES

- [1] M. Afkhami and F. Khosh-Ahang, *Unit graphs of rings of polynomials and power series*, Arab. J. Math. **2** (2013) 233–246.  
doi:10.1007/s40065-013-0067-0
- [2] R. Akhtar, M. Boggess, T. Jackson-Henderson, I. Jimenez, R. Karpman, A. Kinzel and D. Pritikin, *On the Unitary Cayley Graph of a Finite Ring*, Electron. J. Combin. **16** (2009) 1–13.

- [3] M.R. Alfuraidan and Y.F. Zakariya, *Inverse graphs associated with finite groups*, Electron. J. Graph Theory and Appl. **5** (2017) 142–154.  
doi:10.5614/ejgta.2017.5.1.14
- [4] N. Ashrafi, H.R. Maimani, M.R. Pournaki and S. Yassemi, *Unit Graphs Associated with Rings*, Commun. Algebra **38** (2010) 2851–2871.  
doi:10.1080/00927870903095574
- [5] M. Basic and A. Ilic, *Polynomials of Unitary Cayley Graphs*, Filomat **29** (2015) 2079–2086.  
doi:10.2298/FIL1509079B
- [6] J.A. Beachy and W.D. Blair, *Abstract Algebra*, Third Edition (Waveland Press, 2006).
- [7] N. Biggs, *Algebraic Graph Theory*, 2nd Edition (Cambridge India, 2016).
- [8] J.A. Bondy and U.S.R. Murty, *Graph Theory with Application* (Springer India, 2013).
- [9] P. Brizbeitia and R.E. Gaudici, *Counting pure  $k$ -cycles in sequences of Cayley graphs*, Discrete Math. **149** (1996) 11–18.
- [10] T. Chalapathi, RVMSS. Kiran Kumar and F. Smarandache, *Neutrosophic Invertible Graphs of Neutrosophic Rings*, New Trends in Neutrosophic Theory and Applications **2** (2018) 209–217.
- [11] T. Chalapathi and RVMSS. Kiran Kumar, *Self Additive Inverse Elements of Neutrosophic Rings and Fields*, Ann. Pure Appl. Math. **13** (2017) 63–72.  
doi:10.22457/apam.v13n1a7
- [12] T. Chalapathi, S. Sajana and D. Bharathi, *Classical pairs in  $Z_n$* , Notes on Number Theory and Discrete Math. **26** (2020) 59–69.  
doi:10.7546/nntdm.2020.26.1.59-69
- [13] I. Dejter and R.E. Giudici, *On unitary Cayley graphs*, J. Combin. Math. Combin. Comput **18** (1995) 121–124.
- [14] G. Dresden and W.M. Dymacek, *Finding Factors and Factor Rings over the Gaussian Integers*, The Mathematical Association of America **112** (2005) 602–611.  
doi:10.1080/00029890.2005.11920231
- [15] S. Huadong, *A study of unit graphs and unitary cayley graphs associated with rings*, Doctoral Ph.D. Thesis (Memorial University of Newfoundland, 2015).
- [16] D.L. James, *Applications of Unitary Symmetry and Combinatorics* (World Scientific Publishers, 2011).
- [17] W.F. Joe and L.S. Robert, *Rings Generated by Their Units*, J. Algebra **42** (1976) 363–368.
- [18] R. Joy and K. Patra, *Some aspects of Unitary addition Cayley graph of Gaussian integers modulo  $n$* , Matematika **32** (2016) 43–52.  
doi:10.11113/matematika.v32.n1.782

- [19] D. Kiani and M.M.H. Aghaei, *On the Unitary Cayley Graphs of a Ring*, Electron. J. Combin. **19** (2012) 1–10.  
doi:doi.org/10.37236/2214
- [20] W. Klotz and T. Sander, *Some properties of unitary Cayley graphs*, Electron. J. Combin. **14** (2007) 1–12.  
doi:10.37236/963
- [21] X. Liu and S. Zhou, *Spectral Properties of Unitary Cayley Graphs of Finite Commutative Rings*, Electron. J. Combin. **19** (2012) 1–13.
- [22] H.R. Maimani, M.R. Pournaki and S. Yassemi, *Rings which are generated by their units: a graph theoretical approach*, Elem. Math. **65** (2010) 17–25.  
doi:10.4171/EM/134
- [23] R.G. Raphael, *Rings which are generated by their units*, J. Algebra **28** (1974) 199–205.
- [24] I. Shavitt, *Graph Theoretical Concepts for the Unitary Group Approach to the Many-Electron Correlation Problem*, International Journal of Quantum Chemistry: Quantum Chemistry Symposium **11** (1977) 131–148.  
doi:10.1002/qua.560120819
- [25] S.G. Telang, *Number Theory* (Tmh Publisher, 1996).
- [26] A. Tripi, *Cayley Graphs of Groups and Their Applications*, Doctoral Ph.D. Thesis (Missouri State University, 2017).
- [27] I.V. Vitaly, *Introduction to Graph Theory* (Nova Science Publishers. Inc. New York, 2009).

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