

ON BALANCING QUATERNIONS AND LUCAS-BALANCING QUATERNIONS

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Abstract

In this paper we define and study balancing quaternions and Lucas-balancing quaternions. We give the generating functions, matrix generators and Binet formulas for these numbers. Moreover, the well-known properties e.g. Catalan, d' Ocagne identities have been obtained for these quaternions.

Keywords: balancing number, Lucas-balancing number, quaternion, Binet formula, generating function.

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