

## GENERATING FUNCTIONS OF THE PRODUCTS OF BIVARIATE COMPLEX FIBONACCI POLYNOMIALS WITH GAUSSIAN NUMBERS AND POLYNOMIALS

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### Abstract

In this paper, we define and study the bivariate complex Fibonacci and Lucas polynomials. We introduce a operator in order to derive some new symmetric properties of bivariate complex Fibonacci and bivariate complex Lucas polynomials, and give the generating functions of the products of bivariate complex Fibonacci polynomials with Gaussian Fibonacci, Gaussian Lucas and Gaussian Jacobsthal numbers, Gaussian Pell numbers, Gaussian Pell Lucas numbers. By making use of the operator defined in this paper, we give some new generating functions of the products of bivariate complex Fibonacci polynomials with Gaussian Jacobsthal, Gaussian Jacobsthal Lucas polynomials and Gaussian Pell polynomials.

**Keywords:** symmetric functions, generating functions, bivariate complex Fibonacci polynomials, bivariate complex Lucas polynomials.

**2010 Mathematics Subject Classification:** Primary 05E05; Secondary 11B39.

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Received 29 December 2019

Revised 22 February 2020

Accepted 3 July 2020