

## COMMUTATIVITY WITH DERIVATIONS OF SEMIPRIME RINGS

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### Abstract

Let  $R$  be a 2-torsion free semiprime ring with the centre  $Z(R)$ ,  $U$  be a non-zero ideal and  $d: R \rightarrow R$  be a derivation mapping. Suppose that  $R$  admits

- (1) a derivation  $d$  satisfying one of the following conditions:
  - (i)  $[d(x), d(y)] - [x, y] \in Z(R)$  for all  $x, y \in U$ ,
  - (ii)  $[d^2(x), d^2(y)] - [x, y] \in Z(R)$  for all  $x, y \in U$ ,
  - (iii)  $[d(x)^2, d(y)^2] - [x, y] \in Z(R)$  for all  $x, y \in U$ ,
  - (iv)  $[d(x^2), d(y^2)] - [x, y] \in Z(R)$  for all  $x, y \in U$ ,
  - (v)  $[d(x), d(y)] - [x^2, y^2] \in Z(R)$  for all  $x, y \in U$ .
- (2) a non-zero derivation  $d$  satisfying one of the following conditions:
  - (i)  $d([d(x), d(y)]) - [x, y] \in Z(R)$  for all  $x, y \in U$ ,
  - (ii)  $d([d(x), d(y)]) + [x, y] \in Z(R)$  for all  $x, y \in U$ .

Then  $R$  contains a non-zero central ideal.

**Keywords:** semiprime rings, derivations, torsion free rings, central ideal.

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