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THE CAYLEY SUM GRAPH OF IDEALS OF A LATTICE

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Abstract

Let L be a lattice, $\Im(L)$ be the set of ideals of L and S be a subset of $\Im(L)$. In this paper, we introduce an undirected Cayley graph of L, denoted by $\Gamma_{L,S}$ with elements of $\Im(L)$ as the vertex set and, for two distinct vertices I and J, I is adjacent to J if and only if there is an element K of S such that $I \vee K = J$ or $J \vee K = I$. We study some basic properties of the graph $\Gamma_{L,S}$ such as connectivity, girth and clique number. Moreover, we investigate the planarity, outerplanarity and ring graph of $\Gamma_{L,S}$.

Keywords: lattice, Cayley graph, ring graph, outerplanar graph.

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