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ON GENERALIZED DERIVATIONS AND COMMUTATIVITY OF ASSOCIATIVE RINGS

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Abstract

Let $\mathcal{R}$ be a ring with center $Z(\mathcal{R})$. A mapping $f : \mathcal{R} \to \mathcal{R}$ is said to be strong commutativity preserving (SCP) on $\mathcal{R}$ if $[f(x), f(y)] = [x, y]$ and is said to be strong anti-commutativity preserving (SACP) on $\mathcal{R}$ if $f(x) \circ f(y) = x \circ y$ for all $x, y \in \mathcal{R}$. In the present paper, we apply the standard theory of differential identities to characterize SCP and SACP derivations of prime and semiprime rings.

Keywords: generalized derivations, (semi)prime rings, generalized polynomial identities, Martindale ring of quotients.

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