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ANALYTIC PROPERTIES OF THE APOSTOL-VU MULTIPLE FIBONACCI ZETA FUNCTIONS

Utkal Keshari Dutta and Prasanta Kumar Ray

Department of Mathematics, Sambalpur University
Jyoti Vihar, Burla-768019, India

e-mail: utkaldutta@gmail.com
prasantamath@suniv.ac.in

Abstract

In this note we study the analytic continuation of the Apostol-Vu multiple Fibonacci zeta functions

$$\zeta_{AVF,k}(s_1,\ldots,s_k;s_{k+1}) = \sum_{1\le m_1<\cdots<m_k} \frac{1}{F_{m_1}^{s_1}F_{m_2}^{s_2}\cdots F_{m_k}^{s_k}F_{m_1+m_2+\cdots+m_k}^{s_{k+1}}},$$

where $s_1,\ldots,s_{k+1}$ are complex variables and F_n is the n-th Fibonacci number. We find a complete list of poles and their corresponding residues.

Keywords: analytic continuation, Fibonacci numbers, multiple Fibonacci zeta function, Apostol-Vu multiple Fibonacci zeta function.

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