

STRONGLY GENERALIZED RADICAL SUPPLEMENTED MODULES

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Abstract

We introduce and study strongly generalized radical-supplemented modules (or briefly sgrs-modules). With the notation $\text{Rad}_g(R) := \cap\{K : K \leq R_R, K \text{ is both essential and maximal}\}$, we prove that (under some mild conditions on a ring R) every right R -module is a sgrs-module if and only if $\frac{R}{\text{Soc}(R)}$ is right perfect and the idempotents lift module $\text{Rad}_g(R)$.

Keywords: essential submodules, supplemented modules, strongly radical-supplemented modules, (semi-) perfect rings.

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REFERENCES

- [1] F.W. Anderson and K.R. Fuller, *Rings and Categories of Modules* (New York, Springer-Verlag, 1974).
- [2] E. Büyükaşik and E. Türkmen, *Strongly radical supplemented modules*, Ukrainian Math. J. **63** (2011) 1306–1313.
doi:10.1007/s11253-012-0579-3
- [3] J. Clark, C. Lomp, N. Vanaja and R. Wisbauer, *Lifting Modules: Supplements and Projectivity in Module Theory* (Basel, Boston, Berlin, Birkhauser Verlag, 2006).
doi:10.1007/3-7643-7573-6

- [4] B. Koşar, C. Nebiyev and A. Pekin, *A generalization of g-supplemented modules*, Miskolc Math. Notes **20** (2019) 345–352.
doi:10.18514/MMN.2019.2586
- [5] B. Koşar, C. Nebiyev and N. Sökmez, *G-supplemented modules*, Ukrainian Math. J. **67** (2015) 861–864.
doi:10.1007/s11253-015-1127-8
- [6] M.T. Koşan, *δ -lifting and δ -supplemented modules*, Algebra Colloq. **14** (2007) 53–60.
doi:10.1142/S1005386707000065
- [7] C. Lomp, *On semilocal modules and rings*, Comm. Algebra **27** (1999) 1921–1935.
doi:10.1080/00927879908826539
- [8] C. Nebiyev and H. Hüseyin Ökten, *Weakly G-supplemented modules*, European J. Pure Appl. Math. **10** (2017) 521–528.
<https://ejpam.com/index.php/ejpam/article/viewFile/2662/534>
- [9] T.C. Quynh and P.H. Tin, *Some properties of e-supplemented and e-lifting modules*, Vietnam J. Math. **41** (2013) 303–312.
doi:10.1007/s10013-013-0022-6
- [10] L.V. Thuyet and P.H. Tin, *Some characterizations of modules via essentially small sub-modules*, Kyungpook Math. J. **56** (2016) 1069–1083.
doi:10.5666/kmj.2016.56.4.1069
- [11] B.N. Türkmen and E. Türkmen, *On a generalization of weakly supplemented modules*, An. Ştiinț. Univ. Al. I. Cuza Iași. Mat. (N.S.) **63** (2017) 441–448.
doi:10.1515/aicu-2015-0012
- [12] R. Wisbauer, Foundations of Module and Ring Theory (Gordon and Breach, Philadelphia, 1991).
- [13] Y. Zhou, *Generalizations of perfect, semiperfect, and semiregular rings*, Algebra Colloq. **7** (2000) 305–318.
doi:10.1007/s10011-000-0305-9
- [14] D.X. Zhou and X.R. Zhang, *Small-essential submodules and morita duality*, Southeast Asian Bull. Math. **35** (2011) 1051–1062.
<http://www.seams-bull-math.ynu.edu.cn/archive.jsp>
- [15] H. Zöschinger, *Komplementierte moduln über dedekindringen*, J. Algebra **29** (1974) 42–56.
doi:10.1016/0021-8693(74)90109-4

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