

## ON THE GENUS OF THE CO-ANNIHILATING GRAPH OF COMMUTATIVE RINGS

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### Abstract

Let  $R$  be a commutative ring with identity and  $\mathfrak{U}_R$  be the set of all non-zero non-units of  $R$ . The *co-annihilating graph* of  $R$ , denoted by  $\mathcal{CA}_R$ , is a graph with vertex set  $\mathfrak{U}_R$  and two vertices  $x$  and  $y$  are adjacent whenever  $\text{ann}(x) \cap \text{ann}(y) = (0)$ . In this paper, we characterize all commutative Artinian non-local rings  $R$  for which the  $\mathcal{CA}_R$  has genus one and two. Also we characterize all commutative Artinian non-local rings  $R$  for which  $\mathcal{CA}_R$  has crosscap one. Finally, we characterize all finite commutative non-local rings for which  $g(\Gamma_2(R)) = g(\mathcal{CA}_R) = 0$  or  $1$ .

**Keywords:** co-annihilating graph, planar graph, genus; crosscap.

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