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ON EQUALITY OF CERTAIN DERIVATIONS OF LIE ALGEBRAS

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Abstract

Let L be a Lie algebra. A derivation α of L is a commuting derivation (central derivation), if $\alpha(x) \in C_L(x)$ $(\alpha(x) \in Z(L))$ for each $x \in L$. We denote the set of all commuting derivations (central derivations) by $\mathcal{D}(L)$ $(Der_z(L))$. In this paper, first we show $\mathcal{D}(L)$ is subalgebra from derivation algebra L, also we investigate the conditions on the Lie algebra L where commuting derivation is trivial and finally we introduce the family of nilpotent Lie algebras in which $Der_z(L) = \mathcal{D}(L)$.

Keywords: derivation, central derivation, centralizer, commuting derivation.

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