

## **$f$ -FIXED POINTS OF ISOTONE $f$ -DERIVATIONS ON A LATTICE**

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### **Abstract**

In a recent paper, Çeven and Öztürk have generalized the notion of derivation on a lattice to  $f$ -derivation, where  $f$  is a given function of that lattice into itself. Under some conditions, they have characterized the distributive and modular lattices in terms of their isotone  $f$ -derivations. In this paper, we investigate the most important properties of isotone  $f$ -derivations on a lattice, paying particular attention to the lattice (resp. ideal) structures of isotone  $f$ -derivations and the sets of their  $f$ -fixed points. As applications, we provide characterizations of distributive lattices and principal ideals of a lattice in terms of principal  $f$ -derivations.

**Keywords:** lattice, isotone  $f$ -derivation, principal  $f$ -derivation,  $f$ -fixed points set.

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### REFERENCES

- [1] M. Ashraf, S. Ali and C. Haetinger, *On derivations in rings and their applications*, Aligarh Bull. Math. **25** (2006) 79–107.
- [2] G. Birkhoff, *Lattice Theory*, 3rd edition, Amer. Math. Soc. (Providence, RI, 1967).
- [3] Y. Çeven and M. Öztürk, *On  $f$ -derivations of lattices*, Bull. Korean Math. Soc. **45** (2008) 701–707.

- [4] B.A. Davey and H.A. Priestley, *Introduction to Lattices and Order*, 2nd edition (Cambridge University Press, 2002).
- [5] L. Ferrari, *On derivations of lattices*, *Pure Math. and Appl.* **12** (2001) 365–382.
- [6] P. He, X. Xin and J. Zhan, *On derivations and their fixed point sets in residuated lattices*, *Fuzzy Sets and Systems* **303** (2016) 97–113.  
doi:10.1016/j.fss.2016.01.006
- [7] K.H. Kim and B. Davvaz, *On  $f$ -derivations of BE-algebras*, *J. Chungcheong Math. Soc.* **28** (2015) 127–138.  
doi:10.14403/jcms.2015.28.1.127
- [8] B. Kolman, R.C. Busby and S.C. Ross, *Discrete Mathematical Structures*, 4th edition (Prentice Hall PTR, 2000).
- [9] S.M. Lee and K.H. Kim, *A note on  $f$ -derivations of BCC-algebras*, *Pure Math. Sci.* **1** (2012) 87–93.
- [10] Ş.A. Özbal and A. Firat, *On  $f$ -derivations of incline algebras*, *Int. Electronic J. Pure and Appl. Math.* **3** (2011) 83–90.
- [11] M.S. Rao, *Congruences and ideals in a distributive lattice with respect to a derivation*, *Bulletin of the Section of Logic* **42** (2013) 1–10.
- [12] G.C. Rao and K.R. Babu, *The theory of derivations in almost distributive lattice*, *Bulletin of the International Mathematical Virtual Institute* **7** (2017) 203–216.
- [13] S. Roman, *Lattices and Ordered Sets* (Springer Science+Business Media, New York, 2008).
- [14] B.S. Schröder, *Ordered Sets* (Birkhauser, Boston, 2003).
- [15] G. Szász, *Translationen der verbände*, *Acta Fac. Rer. Nat. Univ. Comenianae* **5** (1961) 53–57.
- [16] G. Szász, *Derivations of lattices*, *Acta Sci. Math.* **37** (1975) 149–154.
- [17] J. Wang, Y. Jun, X.L. Xin, T.Y. Li and Y. Zou, *On derivations of bounded hyperlattices*, *J. Math. Res. Appl.* **36** (2016) 151–161.  
doi:10.3770/j.issn:2095-2651.2016.02.003
- [18] Q. Xiao and W. Liu, *On derivations of quantales*, *Open Mathematics* **14** (2016) 338–346.  
doi:10.1515/math-2016-0030
- [19] X.L. Xin, T.Y. Li and J.H. Lu, *On derivations of lattices*, *Information Sciences* **178** (2008) 307–316.  
doi:10.1016/j.ins.2007.08.018
- [20] X.L. Xin, *The fixed set of a derivation in lattices*, *Fixed Point Theory and Applications* **218** (2012) 1–12.  
doi:10.1186/1687-1812-2012-218

- [21] Y.H. Yon and K.H. Kim, *On  $f$ -derivations from semilattices to lattices*, Commun. Korean Math. Soc. **29** (2014) 27–36.  
doi:10.4134/CKMS.2014.29.1.027

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