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RESIDUATED STRUCTURES DERIVED FROM COMMUTATIVE IDEMPOTENT SEMIRINGS

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Abstract

Since the reduct of every residuated lattice is a semiring, we can ask under what condition a semiring can be converted into a residuated lattice. It turns out that this is possible if the semiring in question is commutative, idempotent, G-simple and equipped with an antitone involution. Then the resulting residuated lattice even satisfies the double negation law. Moreover, if the mentioned semiring is finite then it can be converted into a residuated lattice or join-semilattice also without asking an antitone involution on it. To a residuated lattice $\mathbf{L}$ which does not satisfy the double negation law there can be assigned a so-called augmented semiring. This can be used for reconstruction of the so-called core $C(\mathbf{L})$ of $\mathbf{L}$. Conditions under which $C(\mathbf{L})$ constitutes a subuniverse of $\mathbf{L}$ are provided.

Keywords: semiring, commutative, idempotent, G-simple, antitone involution, commutative residuated lattice, commutative residuated join-semilattice, divisible, prelinear, double negation law.

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