

ORDERED REGULAR SEMIGROUPS WITH BIGGEST ASSOCIATES

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Abstract

We investigate the class **BA** of ordered regular semigroups in which each element has a biggest associate $x^\dagger = \max\{y \mid xyx = x\}$. This class properly contains the class **PO** of principally ordered regular semigroups (in which there exists $x^* = \max\{y \mid xyx \leq x\}$) and is properly contained in the class **BI** of ordered regular semigroups in which each element has a biggest inverse x° . We show that several basic properties of the unary operation $x \mapsto x^*$ in **PO** extend to corresponding properties of the unary operation $x \mapsto x^\dagger$ in **BA**. We consider naturally ordered semigroups in **BA** and prove that those that are orthodox contain a biggest idempotent. We determine the structure of some such semigroups in terms of a principal left ideal and a principal right ideal. We also characterise the completely simple members of **BA**. Finally, we consider the naturally ordered semigroups in **BA** that do not have a biggest idempotent.

Keywords: regular semigroup, biggest associate, principally ordered, naturally ordered.

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