

CONRAD'S PARTIAL ORDER ON P.Q.-BAER *-RINGS

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Abstract

We prove that a p.q.-Baer $*$ -ring forms a pseudo lattice with Conrad's partial order and also characterize p.q.-Baer $*$ -rings which are lattices. The initial segments of a p.q.-Baer $*$ -ring with the Conrad's partial order are shown to be an orthomodular posets.

Keywords: Conrad's partial order, p.q.-Baer $*$ -ring, central cover, orthomodular set.

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