

RELATION BETWEEN *BE*-ALGEBRAS AND *g*-HILBERT ALGEBRAS

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Abstract

Hilbert algebras are important tools for certain investigations in algebraic logic since they can be considered as fragments of any propositional logic containing a logical connective implication and the constant 1 which is considered as the logical value “true” and as a generalization of this was defined the notion of *g*-Hilbert algebra. In this paper, we investigate the relationship between *g*-Hilbert algebras, *gi*-algebras, implication groupoid and *BE*-algebras. In fact, we show that every *g*-Hilbert algebra is a self distributive *BE*-algebra and conversely. We show cannot remove the condition self distributivity. Therefore we show that any self distributive commutative *BE*-algebras is a *gi*-algebra and any *gi*-algebra is strong and transitive if and only if it is a commutative *BE*-algebra. We prove that the *MV*-algebra is equivalent to the bounded commutative *BE*-algebra.

Keywords: (Heyting, implication, (*g*)-Hilbert) algebra, *BE/CI*-algebra, dual (*S/Q/BCK*)-algebra, *gi*-algebra, implication groupoid, pre-logic, *MV*-algebra.

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REFERENCES

- [1] J.C. Abbott, *Semi Boolean algebra*, Matem. Vestnik **4** (1967) 177–198.
- [2] A. Borumand Saeid, *CI-algebra is equivalent to dual Q-algebra*, J. Egypt. Math. Soc. **21** (2013) 1–2.
doi:10.1016/j.joems.2012.08.021
- [3] R.A. Borzooei and J. Shohani, *On generalized Hilbert algebras*, Italian Journal of pure and Applied Mathematics **20** (2012) 71–86.
- [4] R.A. Borzooei, A. Borumand Saeid, R. Ameri and A. Rezaei, *Involutory BE-algebras*, J. Math. Appl. **37** (2014) 13–26.
- [5] J. Cirulík, *Subtraction-like operations in nearsemilattices*, Demonstratio Math. **43** (2010) 725–738.
doi:10.1515/dema-2013-0267
- [6] I. Chajda and R. Halaš, *Algebraic properties of pre-logics*, Math. Slovaca **52** (2002) 157–175.
- [7] I. Chajda and R. Halaš, *Distributive implication groupoids*, CEJM **5** (2007) 484–492.
doi:10.2478/s11566-007-0021-5
- [8] C.C. Chang, *Algebraic analysis of many valued logics*, Trans. Amer. Math. Soc. **88** (1958) 467–499.
doi:10.1090/s0002-9947-1958-0094302-9
- [9] W.Y. Chen and J.S. Oliveira, *Implication algebras and the metropolis rota axioms for cubic lattices*, J. Algebra **171** (1995) 383–396.
doi:10.1006/jabr.1995.1017
- [10] A. Diego, *Sur les alg‘ebres de Hilbert*, Collection de Logique Math‘ematique, Serie A 21 (Gauthier-Villars, Paris, 1966).
- [11] W. Dudek, *On ideals in Hilbert algebras*, Acta Univ. Palack. Olom. Fac. Rer. Nat. Mathematica **38** (1999) 31–34.
- [12] R. Halas, *Remarks on commutative Hilbert algebras*, Math. Bohemica **4** (2002) 525–529.
- [13] H.S. Kim and Y.H. Kim, *On BE-algebras*, Sci. Math. Jpn. **66** **1** (2007) 113–116.
- [14] K.H. Kim and Y.H. Yon, *Dual BCK-algebra and MV-algebra, On algebras with a generalized implication*, Sci. Math. Jpn. (2007) 393–399.
- [15] K.J. Lee, Y.B. Jun and Y.H. Kim, *Weak forms of subtraction algebras*, Bull. Korean Math. Soc. **45** (2008) 437–444.
doi:10.4134/BKMS.2008.45.3.437
- [16] B.L. Meng, *CI-algebras*, Sci. Math. Jpn. **71** (2010) 695–701.
- [17] A. Rezaei and A. Borumand Saeid, *Some results in BE-algebras*, An. Univ. Oradea, Fasc. Mat. Tom XIX. (2012) 33–44.

- [18] A. Rezaei and A. Borumand Saeid, *Commutative ideals in BE-algebras*, Kyungpook Math. J. **52** (2012) 483–494.
doi:10.5666/KMJ.2012.52.4.483
- [19] A. Rezaei, A. Borumand Saeid and R.A. Borzooei, *Relation between Hilbert algebras and BE-algebras*, Appl. Math. **8** (2013) 573–584.
- [20] A. Rezaei and A. Borumand Saeid, *Relation between dual S-algebras and BE-algebras*, LE Matematiche. Vol. LXX (2015) Fasc. I, pp. 71–79.
- [21] A. Walendziak, *On commutative BE-algebras*, Sci. Math. Jpn. **69** (2008) 585–588.
- [22] Y.H. Yon and E.A. Choi, *Heyting algebra and t-algebra*, Bulletin of the Chungcheong Mathematical Society **11** (1998) 13–26.
- [23] Y.H. Yon and K.H. Kim, *On algebras with a generalized implication*, Math. Slovaca **63** (2013) 947–958.
doi:10.2478/s12175-013-01476-x
- [24] B. Zelinka, *Subtraction semigroup*, Math. Bohemica **120** (1995) 445–447.

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