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CONGRUENCES AND TRAJECTORIES IN PLANAR SEMIMODULAR LATTICES

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Abstract

A 1955 result of J. Jakubík states that for the prime intervals $\mathfrak{p}$ and $\mathfrak{q}$ of a finite lattice, $\mathrm{con}(\mathfrak{p}) \geq \mathrm{con}(\mathfrak{q})$ iff $\mathfrak{p}$ is congruence-projective to $\mathfrak{q}$ (*via* intervals of arbitrary size). The problem is how to determine whether $\mathrm{con}(\mathfrak{p}) \geq \mathrm{con}(\mathfrak{q})$ involving only prime intervals.

Two recent papers approached this problem in different ways. G. Czédli's used trajectories for slim rectangular lattices—a special subclass of slim, planar, semimodular lattices. I used the concept of prime-projectivity for arbitrary finite lattices. In this note I show how my approach can be used to reprove Czédli's result and generalize it to arbitrary slim, planar, semimodular lattices.

Keywords: semimodular lattice, planar lattice, slim lattice, rectangular lattice, congruence, trajectory, prime interval.

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