

GENERALIZED CHEBYSHEV POLYNOMIALS

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Abstract

Let $h(x)$ be a non constant polynomial with rational coefficients. Our aim is to introduce the $h(x)$ -Chebyshev polynomials of the first and second kind T_n and U_n . We show that they are in a \mathbb{Q} -vectorial subspace $E_n(x)$ of $\mathbb{Q}[x]$ of dimension n . We establish that the polynomial sequences $(h^k T_{n-k})_k$ and $(h^k U_{n-k})_k$, $(0 \leq k \leq n-1)$ are two bases of $E_n(x)$ for which T_n and U_n admit remarkable integer coordinates.

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