

## GENERALIZED DERIVATIONS WITH LEFT ANNIHILATOR CONDITIONS IN PRIME AND SEMIPRIME RINGS

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### Abstract

Let  $R$  be a prime ring with its Utumi ring of quotients  $U$ ,  $C = Z(U)$  be the extended centroid of  $R$ ,  $H$  and  $G$  two generalized derivations of  $R$ ,  $L$  a noncentral Lie ideal of  $R$ ,  $I$  a nonzero ideal of  $R$ . The left annihilator of  $S \subseteq R$  is denoted by  $l_R(S)$  and defined by  $l_R(S) = \{x \in R \mid xS = 0\}$ . Suppose that  $S = \{H(u^n)u^n + u^nG(u^n) \mid u \in L\}$  and  $T = \{H(x^n)x^n + x^nG(x^n) \mid x \in I\}$ , where  $n \geq 1$  is a fixed integer. In the paper, we investigate the cases when the sets  $l_R(S)$  and  $l_R(T)$  are nonzero.

**Keywords:** prime ring, derivation, Lie ideal, generalized derivation, extended centroid, Utumi quotient ring.

**2010 Mathematics Subject Classification:** 16W25, 16W80, 16N60.

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Received 27 October 2016

Revised 1 June 2017

Accepted 6 July 2017

