

## ZERO-DIVISOR GRAPHS OF REDUCED RICKART \*-RINGS

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### Abstract

For a ring  $A$  with an involution  $*$ , the *zero-divisor graph* of  $A$ ,  $\Gamma^*(A)$ , is the graph whose vertices are the nonzero left zero-divisors in  $A$  such that distinct vertices  $x$  and  $y$  are adjacent if and only if  $xy^* = 0$ . In this paper, we study the zero-divisor graph of a Rickart  $*$ -ring having no nonzero nilpotent element. The distance, diameter, and cycles of  $\Gamma^*(A)$  are characterized in terms of the collection of prime strict ideals of  $A$ . In fact, we prove that the clique number of  $\Gamma^*(A)$  coincides with the cellularity of the hull-kernel topological space  $\Sigma(A)$  of the set of prime strict ideals of  $A$ , where cellularity of the topological space is the smallest cardinal number  $m$  such that every family of pairwise disjoint non-empty open subsets of the space have cardinality at most  $m$ .

**Keywords:** reduced ring, Rickart  $*$ -ring, zero-divisor graph, prime strict ideals.

**2010 Mathematics Subject Classification:** Primary: 16W10, Secondary: 05C25, 05C15.

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Received 28 June 2016  
1st Revised 17 October 2016  
2st Revised 23 January 2017

