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GENERALIZED PELL EQUATIONS FOR 2 × 2 MATRICES

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Abstract

In this paper we consider the solutions of the generalized matrix Pell equations $X^2 - dY^2 = cI$, where X and Y are 2×2 matrices over $\mathbb{Z}$, d is a non-zero (positive or negative) square-free integer, c is an arbitrary integer and I is the 2×2 identity matrix. We determine all solutions of such equations for $c = \pm 1$, as well as all non-commutative solutions for an arbitrary c.

Keywords: matrix equations, Pell equation.

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