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VARIETIES OF REGULAR ALGEBRAS AND UNRANKED TREE LANGUAGES

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Abstract

In this paper we develop a variety theory for unranked tree languages and unranked algebras. In an unranked tree any symbol may label a node with any number of successors. Such trees appear in markup languages such as XML and as syntactic descriptions of natural languages. In the corresponding algebras each operation is defined for any number of arguments, but in the regular algebras used as tree recognizers the operations are finite-state computable. We develop the basic theory of regular algebras for a setting in which algebras over different operator alphabets are considered together. Using syntactic algebras of unranked tree languages we establish a bijection between varieties of unranked tree languages and varieties of regular algebras. As varieties of unranked tree languages are usually defined by means of congruences of term algebras, we introduce also varieties of congruences and a general device for defining such varieties. Finally, we show that the natural unranked counterparts of several varieties of ranked tree languages form varieties in our sense.

Keywords: unranked algebras, unranked trees, regular algebras, syntactic algebras, varieties of tree languages, varieties of regular algebras.

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