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ON THE GENUS OF THE CAYLEY GRAPH OF A COMMUTATIVE RING

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Abstract

Let R be a commutative ring with non-zero identity and let $Z(R)$ be the set of all zero-divisors. The Cayley graph $\mathbb{CAY}(R)$ of R is the simple undirected graph whose vertices are elements of R and two distinct vertices x and y are joined by an edge if and only if $x - y \in Z(R)$. In this paper, we determine all isomorphism classes of finite commutative rings with identity whose $\mathbb{CAY}(R)$ has genus one.

Keywords: Cayley graph, local ring, zero-divisor, planar graph, genus.

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