Discussiones Mathematicae
General Algebra and Applications 36 (2016) 59–60
doi:10.7151/dmgaa.1248

Full PDF
DM-GAA Page

QUASIORDER LATTICES ARE FIVE-GENERATED

Júlia Kulin

University of Szeged, Bolyai Institute
Szeged, Aradi vértanúk tere 1, Hungary 6720

e-mail: kulin@math.u-szeged.hu

Abstract

A quasiorder (relation), also known as a preorder, is a reflexive and transitive relation. The quasiorders on a set A form a complete lattice with respect to set inclusion. Assume that A is a set such that there is no inaccessible cardinal less than or equal to $|A|$; note that in Kuratowski's model of ZFC, all sets A satisfy this assumption. Generalizing the 1996 result of Ivan Chajda and Gábor Czédli, also Tamás Dolgos' recent achievement, we prove that in this case the lattice of quasiorders on A is five-generated, as a complete lattice.

Keywords: quasiorder lattice, preorder lattice, accessible cardinal.

2010 Mathematics Subject Classification: Primary: 06B99, Secondary: 06B23.

References

[1] G. Czédli, *A Horn sentence for involution lattices of quasiorders*, Order **11** (1994) 391–395. doi:10.1007/BF01108770

[2] I. Chajda and G. Czédli, *How to generate the involution lattice of quasiorders?*, Studia Sci. Math. Hungar. **32** (1996) 415–427.

[3] G. Czédli, *Four-generated large equivalence lattices*, Acta Sci. Math. **62** (1996) 47–69.

[4] G. Czédli, *Lattice generation of small equivalences of a countable set*, Order **13** (1996) 11–16. doi:10.1007/BF00383964

[5] G. Czédli, $(1+1+2)$*-generated equivalence lattices*, J. Algebra **221** (1999) 439–462. doi:10.1006/jabr.1999.8003

This research was supported by NFSR of Hungary (OTKA), grant number K 115518, and by the European Union under the project TÁMOP-4.2.2.B-15/1/KONV-2015-0006.

[6] T. Dolgos, *Generating equivalence and quasiorder lattices over finite sets* (in Hungarian) BSc Thesis, University of Szeged (2015).

[7] K. Kuratowski, *Sur l'état actuel de l'axiomatique de la théorie des ensembles*, Ann. Soc. Polon. Math. **3** (1925) 146–147.

[8] A. Levy, Basic Set Theory (Springer-Verlag, Berlin-Heidelberg-New York, 1979).

[9] H. Strietz, *Finite partition lattices are four-generated*, Proc. Lattice Th. Conf. Ulm (1975) 257–259.

[10] H. Strietz, *Über Erzeugendenmengen endlicher Partitionverbände*, Studia Sci. Math. Hungar. **12** (1977) 1–17.

[11] G. Takách, *Three-generated quasiorder lattices*, Discuss. Math. Algebra and Stochastic Methods **16** (1996) 81–98.

[12] J. Tůma, *On the structure of quasi-ordering lattices*, Acta Universitatis Carolinae, Mathematica et Physica **43** (2002) 65–74.

[13] L. Zádori, *Generation of finite partition lattices*, Lectures in Universal Algebra, Colloquia Math. Soc. J. Bolyai **43** Proc. Conf. Szeged (1983) 573–586 (North Holland, Amsterdam-Oxford-New York, 1986).

Received 29 October 2015
Revised 5 November 2015