

POINTED PRINCIPALLY ORDERED REGULAR SEMIGROUPS

T.S. BLYTH

Mathematical Institute
University of St Andrews, Scotland

e-mail: tsb@st-and.ac.uk

AND

G.A. PINTO

School of Mathematics, Physics and Technology
College of The Bahamas, Freeport, Commonwealth of The Bahamas

e-mail: gcapinto@gmail.com

Abstract

An ordered semigroup S is said to be *principally ordered* if, for every $x \in S$ there exists $x^* = \max \{y \in S \mid xyx \leq x\}$. Here we investigate those principally ordered regular semigroups that are *pointed* in the sense that the classes modulo Green's relations \mathcal{L} , \mathcal{R} , \mathcal{D} have biggest elements which are idempotent. Such a semigroup is necessarily a semiband. In particular we describe the subalgebra of $(S; *)$ generated by a pair of comparable idempotents that are \mathcal{D} -related. We also prove that those \mathcal{D} -classes which are subsemigroups are ordered rectangular bands.

Keywords: regular semigroup, principally ordered, naturally ordered, Green's relations.

2010 Mathematics Subject Classification: 06F05, 20M17.

REFERENCES

- [1] T.S. Blyth, *Lattices and Ordered Algebraic Structures* (Springer, 2005). doi:10.1007/b139095
- [2] T.S. Blyth and M.F. Janowitz, *Residuation Theory* (Pergamon, 1972).
- [3] T.S. Blyth and G.A. Pinto, *Principally ordered regular semigroups*, Glasgow Math. J. **32** (1990) 349–364. doi:10.1017/S0017089500009435

- [4] T.S. Blyth and G.A. Pinto, *Idempotents in principally ordered regular semigroups*, Communications in Algebra **19** (1991) 1549–1563. doi:10.1080/00927879108824220
- [5] T.S. Blyth and M.H. Almeida Santos, *On weakly multiplicative inverse transversals*, Proc. Edinburgh Math. Soc. **37** (1993) 93–99. doi:10.1017/S001309150001871X
- [6] P.M. Higgins, Techniques of Semigroup Theory (Oxford Science Publications, 1992). doi:10.1007/BF02573500

Received 11 January 2016

Revised 20 May 2016