

## GRADED HILBERT-SYMBOL EQUIVALENCE OF NUMBER FIELDS

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### Abstract

We present a new criterion for the existence of Hilbert-symbol equivalence of two number fields. In principle, we show that the system of local conditions for this equivalence may be expressed in terms of Clifford invariants in place of Hilbert-symbols, shifting the focus from Brauer groups to Brauer-Wall groups.

**Keywords:** Brauer group, Brauer-Wall group, Hilbert symbol equivalence, Witt equivalence, graded quaternion algebras.

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