

JORDAN NUMBERS, STIRLING NUMBERS AND SUMS OF POWERS

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Abstract

In the paper a new combinatorical interpretation of the Jordan numbers is presented. Binomial type formulae connecting both kinds of numbers mentioned in the title are given. The decomposition of the product of polynomial of variable n into the sums of k th powers of consecutive integers from 1 to n is also studied.

Keywords: Bernoulli numbers, binomial coefficients, Jordan numbers, Stirling numbers, Živković numbers.

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