

## STRONG QUASI $k$ -IDEALS AND THE LATTICE DECOMPOSITIONS OF SEMIRINGS WITH SEMILATTICE ADDITIVE REDUCT

ANJAN KUMAR BHUNIYA

*Department of Mathematics, Visva-Bharati,  
Santiniketan-731235, India*

e-mail: anjankbhuniya@gmail.com

AND

KANCHAN JANA

*Department of Mathematics, Katwa College,  
Katwa-713130, India*

e-mail: kjana76@gmail.com

### Abstract

Here we introduce the notion of strong quasi  $k$ -ideals of a semiring in  $SL^+$  and characterize the semirings that are distributive lattices of  $t$ - $k$ -simple( $t$ - $k$ -Archimedean) subsemirings by their strong quasi  $k$ -ideals. A quasi  $k$ -ideal  $Q$  is strong if it is an intersection of a left  $k$ -ideal and a right  $k$ -ideal. A semiring  $S$  in  $SL^+$  is a distributive lattice of  $t$ - $k$ -simple semirings if and only if every strong quasi  $k$ -ideal is a completely semiprime  $k$ -ideal of  $S$ . Again  $S$  is a distributive lattice of  $t$ - $k$ -Archimedean semirings if and only if  $\sqrt{Q}$  is a  $k$ -ideal, for every strong quasi  $k$ -ideal  $Q$  of  $S$ .

**Keywords:** quasi  $k$ -ideal, strong quasi  $k$ -ideal, strong quasi  $k$ -simple,  $t$ - $k$ -simple,  $t$ - $k$ -Archimedean.

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