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GENERALIZED DERIVATIONS IN PRIME RINGS AND BANACH ALGEBRAS

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Abstract

Let R be a prime ring with extended centroid C, F a generalized derivation of R and $n \geq 1$, $m \geq 1$ fixed integers. In this paper we study the situations:

1. $(F(x \circ y))^m = (x \circ y)^n$ for all $x, y \in I$, where I is a nonzero ideal of R;
2. $(F(x \circ y))^n = (x \circ y)^n$ for all $x, y \in I$, where I is a nonzero right ideal of R.

Moreover, we also investigate the situation in semiprime rings and Banach algebras.

Keywords: prime ring, generalized derivation, extended centroid, Utumi quotient ring.

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