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GENERALIZED DERIVATIONS IN PRIME RINGS AND BANACH ALGEBRAS

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Abstract

Let R be a prime ring with extended centroid C, F a generalized derivation of R and $n \geq 1$, $m \geq 1$ fixed integers. In this paper we study the situations:

1. $(F(x \circ y))^m = (x \circ y)^n$ for all $x, y \in I$, where I is a nonzero ideal of R;
2. $(F(x \circ y))^n = (x \circ y)^n$ for all $x, y \in I$, where I is a nonzero right ideal of R.

Moreover, we also investigate the situation in semiprime rings and Banach algebras.

Keywords: prime ring, generalized derivation, extended centroid, Utumi quotient ring.

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References

[1] N. Argac and H.G. Inceboz, *Derivations of prime and semiprime rings*, J. Korean Math. Soc. **46** (2009) 997–1005. doi:10.4134/JKMS.2009.46.5.997

[2] M. Ashraf and N. Rehman, *On commutativity of rings with derivations*, Result. Math. **42** (2002) 3–8. doi:10.1007/BF03323547

[3] K.I. Beidar, *Rings of quotients of semiprime rings*, Vestnik Moskov. Univ. Ser I Math. Meh. (Engl. Transl:. Moscow Univ. Math. Bull.) **33** (1978) 36–42.

[4] M. Brešar and J. Vukman, *On left derivations and related mappings*, Proc. Amer. Math. Soc. **110** (1990) 7–16. doi:10.1090/S0002-9939-1990-1028284-3

[5] M. Brešar, *On the distance of the composition of the two derivations to be the generalized derivations*, Glasgow Math. J. **33** (1991) 89–93. doi:10.1017/S0017089500008077

[6] C.M. Chang, *Power central values of derivations on multilinear polynomials*, Taiwanese J. Math. **7** (2003) 329–338.

[7] C.L. Chuang, *GPIs having coefficients in Utumi quotient rings*, Proc. Amer. Math. Soc. **103** (1988) 723–728. doi:10.1090/S0002-9939-1988-0947646-4

[8] M.N. Daif and H.E. Bell, *Remarks on derivations on semiprime rings*, Internat. J. Math. & Math. Sci. **15** (1992) 205–206. doi:10.1155/S0161171292000255

[9] V. De Filippis and S. Huang, *Generalized derivations on semi prime rings*, Bull. Korean Math. Soc. **48** (2011) 1253–1259. doi:10.4134/BKMS.2011.48.6.1253

[10] B. Dhara, *Remarks on generalized derivations in prime and semiprime rings*, Internat. J. Math. & Math. Sc. **2010** (Article ID 646587) 6 pages.

[11] T.S. Erickson, W.S. Martindale III and J.M. Osborn, *Prime nonassociative algebras*, Pacific J. Math. **60** (1975) 49–63. doi:10.2140/pjm.1975.60.49

[12] C. Faith and Y. Utumi, *On a new proof of Litoff's theorem*, Acta Math. Acad. Sci. Hung. **14** (1963) 369–371. doi:10.1007/BF01895723

[13] I.N. Herstein, Topics in ring theory (Univ. of Chicago Press, Chicago, 1969).

[14] S. Huang and B. Davvaz, *Generalized derivations of rings and Banach algebras*, Comm. Algebra **41** (2013) 1188–1194. doi:10.1080/00927872.2011.642043

[15] S. Huang, *On generalized derivations of prime and semiprime rings*, Taiwanese J. Math. **16** (2012) 771–776.

[16] N. Jacobson, Structure of rings (Amer. Math. Soc. Colloq. Pub. 37. Providence, RI: Amer. Math. Soc., 1964).

[17] B.E. Johnson and A.M. Sinclair, *Continuity of derivations and a problem of Kaplansky*, Amer. J. Math. **90** (1968) 1067–1073. doi:10.2307/2373290

[18] V.K. Kharchenko, *Differential identity of prime rings*, Algebra and Logic **17** (1978) 155–168. doi:10.1007/BF01670115

[19] B. Kim, *On the derivations of semiprime rings and noncommutative Banach algebras*, Acta Math. Sinica **16** (2000) 21–28. doi:10.1007/s101149900020

[20] C. Lanski, *An engel condition with derivation*, Proc. Amer. Math. Soc. **118** (1993) 731–734. doi:10.1090/S0002-9939-1993-1132851-9

[21] T.K. Lee, *Semiprime rings with differential identities*, Bull. Inst. Math. Acad. Sinica **20** (1992) 27–38.

[22] T.K. Lee, *Generalized derivations of left faithful rings*, Comm. Algebra **27** (1999) 4057–4073. doi:10.1080/00927879908826682

[23] W.S. Martindale III, *Prime rings satisfying a generalized polynomial identity*, J. Algebra **12** (1969) 576–584. doi:10.1016/0021-8693(69)90029-5

[24] M. Mathieu, *Properties of the product of two derivations of a C^*-algebra*, Canad. Math. Bull. **32** (1989) 490–497. doi:10.4153/CMB-1989-072-4

[25] M.A. Quadri, M.S. Khan and N. Rehman, *Generalized derivations and commutativity of prime rings*, Indian J. Pure Appl. Math. **34** (2003) 1393–1396.

[26] A.M. Sinclair, *Continuous derivations on Banach algebras*, Proc. Amer. Math. Soc. **20** (1969) 166–170. doi:10.1090/S0002-9939-1969-0233207-X

[27] I.M. Singer and J. Wermer, *Derivations on commutative normed algebras*, Math. Ann. **129** (1955) 260–264. doi:10.1007/BF01362370

[28] M. Thomas, *The image of a derivation is contained in the radical*, Ann. Math. **128** (1988) 435–460. doi:10.2307/1971432

[29] J. Vukman, *On derivations in prime rings and Banach algebras*, Proc. Amer. Math. Soc. **116** (1992) 877–884.

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