

NIL-EXTENSIONS OF COMPLETELY SIMPLE SEMIRINGS

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Abstract

A semiring S is said to be a quasi completely regular semiring if for any $a \in S$ there exists a positive integer n such that na is completely regular. The present paper is devoted to the study of completely Archimedean semirings. We show that a semiring S is a completely Archimedean semiring if and only if it is a nil-extension of a completely simple semiring. This result extends the crucial structure theorem of completely Archimedean semigroup.

Keywords: ideal extension, nil-extension, bi-ideal, completely Archimedean semirings, completely simple semiring.

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