Discussiones Mathematicae
General Algebra and Applications 33 (2013) 13–16
doi:10.7151/dmgaa.1194

THE EXOCENTER AND TYPE DECOMPOSITION OF A GENERALIZED PSEUDOEFFECT ALGEBRA

David J. Foulis

Department of Mathematics an Statistics
Univ. of Massachusetts, Amherst, MA, USA

e-mail: foulis@math.umass.edu

Silvia Pulmannová

Mathematical Institute, Slovak Academy of Sciences
Štefánikova 49, 814 73 Bratislava, Slovakia

e-mail: pulmann@mat.savba.sk

and

Elena Vinceková

Mathematical Institute, Slovak Academy of Sciences
Štefánikova 49, 814 73 Bratislava, Slovakia

e-mail: vincekova@mat.savba.sk

Abstract

We extend the notion of the exocenter of a generalized effect algebra (GEA) to a generalized pseudoeffect algebra (GPEA) and show that elements of the exocenter are in one-to-one correspondence with direct decompositions of the GPEA; thus the exocenter is a generalization of the center of a pseudoeffect algebra (PEA). The exocenter forms a boolean algebra and the central elements of the GPEA correspond to elements of a sublattice of the exocenter which forms a generalized boolean algebra. We extend the notion of central orthocompleteness to GPEA, prove that the exocenter of a centrally orthocomplete GPEA (COGPEA) is a complete boolean algebra and show that the sublattice corresponding to the center is a complete boolean subalgebra. We also show that in a COGPEA, every element admits an exocentral cover and that the family of all exocentral covers, the so-called exocentral cover system, has the properties of a hull system on a generalized effect algebra. We extend the notion of type determining (TD) sets, originally introduced for effect algebras and then extended to GEAs and PEAs, to GPEAs, and prove a type-decomposition theorem, analogous to the type decomposition of von Neumann algebras.

Keywords: pseudoeffect algebra, generalized pseudoeffect algebra, center, exocenter, central orthocompleteness, type determining set, type decomposition.

2010 Mathematics Subject Classification: Primary: 81P10, 08A55, Secondary: 03G12.

References

[1] P. Bush, P. Lahti and P. Mittelstaedt, The Quantum Theory of Measurement (Lecture Notes in Physics, Springer, Berlin-Heidelberg-New York, 1991).

[2] J.C Carrega, G. Chevalier and R. Mayet, *Direct decomposition of orthomodular lattices*, Alg. Univers. **27** (1990) 480–496. doi:10.1007/DF01188994

[3] A. Dvurečenskij, *Central elements and Cantor-Bernstein's theorem for pseudo-effect algebras*, J. Aust. Math. Soc. **74** (2003) 121–143.

[4] A. Dvurečenskij and S. Pulmannová, New Trends in Quantum Structures (Kluwer, Dordrecht, 2000).

[5] A. Dvurečenskij and T. Vetterlein, *Pseudoeffect algebras I. Basic properties*, Int. J. Theor. Phys. **40** (2001) 685–701. doi:10.1023/A:1004192715509

[6] A. Dvurečenskij and T. Vetterlein, *Pseudoeffect algebras II. Group representations*, Int. J. Theor. Phys. **40** (2001) 703–726. doi:10.1023/A:1004144832348

[7] A. Dvurečenskij and T. Vetterlein, *Algebras in the positive cone of po-groups*, Order **19** (2002) 127–146. doi:10.1023A:1016551707-476

[8] A. Dvurečenskij and T. Vetterlein, *Generalized pseudo-effect algebras*, in: Lectures on Soft Computing and Fuzzy Logic; Adv. Soft Comput, (Ed(s)), (Physica, Heidelberg, 2001) 89–111.

[9] D.J. Foulis and M.K. Bennett, *Effect algebras and unsharp quantum logics*, Found. Phys. **24** (1994) 1331–1352. doi:10.1007/BF02283036

[10] D.J. Foulis and S. Pulmannová, *Type-decomposition of an effect algebra*, Found. Phys. **40** (2010) 1543–1565. doi:10.1007/s10701-009-9344-3

[11] D.J Foulis and S. Pulmannová, *Centrally orthocomplete effect algebras*, Algebra Univ. **64** (2010) 283–307. doi:10.1007/s00012-010-0100-5

[12] D.J. Foulis and S. Pulmannová, *Hull mappings and dimension effect algebras*, Math. Slovaca **61** (2011) 485–522. doi:10.2478/s12175-011-0025-2

[13] D.J. Foulis and S. Pulmannová, *The exocenter of a generalized effect algebra*, Rep. Math. Phys. **61** (2011) 347–371.

[14] D.J. Foulis and S. Pulmannová, *The center of a generalized effect algebra*, to appear in Demonstratio Math.

[15] D.J. Foulis and S. Pulmannová, *Hull determination and type decomposition for a generalized effect algebra*, Algebra Univers. **69** (2013) 45–81. doi:10.1007/s00012-012-0214-z

[16] D.J. Foulis, S. Pulmannová and E. Vinceková, *Type decomposition of a pseudoeffect algebra*, J. Aust. Math. Soc. **89** (2010) 335–358. doi:10.1017/S1446788711001042

[17] K.R. Goodearl and F. Wehrung, *The Complete Dimension Theory of Partially Ordered Systems with Equivalence and Orthogonality*, Mem. Amer. Math. Soc. **831** (2005).

[18] R.J. Greechie, D.J. Foulis and S. Pulmannová, *The center of an effect algebra*, Order **12** (1995) 91–106. doi:10.1007/BF01108592

[19] G. Grätzer, General Lattice Theory (Academic Press, New York, 1978).

[20] J. Hedlíková and S. Pulmannová, *Generalized difference posets and orthoalgebras*, Acta Math. Univ. Comenianae **45** (1996) 247–279.

[21] M.F. Janowitz, *A note on generalized orthomodular lattices*, J. Natur. Sci and Math. **8** (1968) 89–94.

[22] G. Jenča, *Subcentral ideals in generalized effect algebras*, Int. J. Theor. Phys. **39** (2000) 745–755. doi:10.1023?A:1003610426013

[23] G. Kalmbach, Measures and Hilbert Lattices (World Scientific Publishing Co., Singapore, 1986).

[24] G. Kalmbach and Z. Riečanová, *An axiomatization for abelian relative inverses*, Demonstratio Math. **27** (1994) 535–537.

[25] F. Kôpka and F. Chovanec, *D-posets*, Math. Slovaca **44** (1994) 21–34.

[26] L.H. Loomis, The Lattice-Theoretic Background of the Dimension Theory of Operator Algebras (Mem. Amer. Math. Soc. No. 18, 1955).

[27] S. Maeda, *Dimension functions on certain general lattices*, J. Sci. Hiroshima Univ **A 19** (1955) 211–237.

[28] A. Mayet-Ippolito, *Generalized orthomodular posets*, Demonstratio Math. **24** (1991) 263–274.

[29] F.J. Murray and J. von Neumann, On Rings of Operators, J. von Neumann colected works, vol. III, Pergamon Press, Oxford, 1961, 6–321.

[30] S. Pulmannová and E. Vinceková, *Riesz ideals in generalized effect algebras and in their unitizations*, Algebra Univ. **57** (2007) 393–417. doi:10.1007/s00012-007-2043-z

[31] S. Pulmannová and E. Vinceková, *Abelian extensions of partially ordered partial monoids*, Soft Comput. **16** (2012) 1339–1346. doi:10.1007/s00500-012-0814-8

[32] A. Ramsay, *Dimension theory in complete weakly modular orthocomplemented lattices*, Trans. Amer. Math. Soc. **116** (1965) 9–31.

[33] Z. Riečanová, *Subalgebras, intervals, and central elements of generalized effect algebras*, Int. J. Theor. Phys. **38** (1999) 3209–3220. doi:10.1023/A:1026682215765

[34] M.H. Stone, *Postulates for Boolean algebras and generalized Boolean algebras*, Amer. J. Math. **57** (1935) 703–732.

[35] A. Wilce, *Perspectivity and congruence in partial Abelian semigroups*, Math. Slovaca **48** (1998) 117–135.

[36] Y. Xie and Y. Li, *Riesz ideals in generalized pseudo effect algebras and in their unitizations*, Soft Comput. **14** (2010) 387–398. doi:10.1007/s00500-009-0412-6

Received 19 October 2012
Revised 18 December 2012