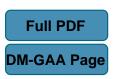
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## THE MONOID OF GENERALIZED HYPERSUBSTITUTIONS OF TYPE au = (n)

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## Abstract

A (usual) hypersubstitution of type  $\tau$  is a function which takes each operation symbol of the type to a term of the type, of the same arity. The set of all hypersubstitutions of a fixed type  $\tau$  forms a monoid under composition, and semigroup properties of this monoid have been studied by a number of authors. In particular, idempotent and regular elements, and the Green's relations, have been studied for type (n) by S.L. Wismath.

A generalized hypersubstitution of type  $\tau = (n)$  is a mapping  $\sigma$ which takes the *n*-ary operation symbol f to a term  $\sigma(f)$  which does not necessarily preserve the arity. Any such  $\sigma$  can be inductively extended to a map  $\hat{\sigma}$  on the set of all terms of type  $\tau = (n)$ , and any two such extensions can be composed in a natural way. Thus, the set  $Hyp_G(n)$  of all generalized hypersubstitutions of type  $\tau = (n)$  forms a monoid. In this paper we study the semigroup properties of  $Hyp_G(n)$ .

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In particular, we characterize the idempotent and regular generalized hypersubstitutions, and describe some classes under Green's relations of this monoid.

**Keywords:** monoid, regular elements, idempotent elements, Green's relations, generalized hypersubstitution.

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