

## THE MONOID OF GENERALIZED HYPERSUBSTITUTIONS OF TYPE $\tau = (n)$

WATTAPONG PUNINAGOO

AND

SORASAK LEERATANAVALLEE\*

*Department of Mathematics,  
Faculty of Science, Chiang Mai University,  
Chiang Mai 50200, Thailand*

**e-mail:** wattapong1p@yahoo.com

**e-mail:** scislrtt@chiangmai.ac.th

### Abstract

A (usual) hypersubstitution of type  $\tau$  is a function which takes each operation symbol of the type to a term of the type, of the same arity. The set of all hypersubstitutions of a fixed type  $\tau$  forms a monoid under composition, and semigroup properties of this monoid have been studied by a number of authors. In particular, idempotent and regular elements, and the Green's relations, have been studied for type  $(n)$  by S.L. Wismath.

A generalized hypersubstitution of type  $\tau = (n)$  is a mapping  $\sigma$  which takes the  $n$ -ary operation symbol  $f$  to a term  $\sigma(f)$  which does not necessarily preserve the arity. Any such  $\sigma$  can be inductively extended to a map  $\hat{\sigma}$  on the set of all terms of type  $\tau = (n)$ , and any two such extensions can be composed in a natural way. Thus, the set  $Hyp_G(n)$  of all generalized hypersubstitutions of type  $\tau = (n)$  forms a monoid. In this paper we study the semigroup properties of  $Hyp_G(n)$ .

---

\*Corresponding author.

In particular, we characterize the idempotent and regular generalized hypersubstitutions, and describe some classes under Green's relations of this monoid.

**Keywords:** monoid, regular elements, idempotent elements, Green's relations, generalized hypersubstitution.

**2000 Mathematics Subject Classification:** 20M05, 20M99, 20N02.

#### REFERENCES

- [1] K. Denecke, D. Lau, R. Pöschel and D. Schweigert, *Hyperidentities, Hyper-equational Classes, and Clone Congruences*, Verlag Hölder-Pichler-Tempsky, Wien, Contributions to General Algebra **7** (1991), 97–118.
- [2] S. Leeratanavalee and K. Denecke, *Generalized Hypersubstitutions and Strongly Solid Varieties*, p. 135–145 in: General Algebra and Applications, Proc. of the “59<sup>th</sup> Workshop on General Algebra”, ‘15<sup>th</sup> Conference for Young Algebraists Potsdam 2000’, Shaker Verlag 2000.
- [3] S. Leeratanavalee, *Submonoids of Generalized Hypersubstitutions*, Demonstratio Mathematica **XL** (1) (2007), 13–22.
- [4] W. Puninagool and S. Leeratanavalee, *All Regular Elements in  $\text{Hyp}_G(2)$* , preprint 2009.
- [5] W. Puninagool and S. Leeratanavalee, *Green's Relations on  $\text{Hyp}_G(2)$* , preprint 2009.
- [6] W. Puninagool and S. Leeratanavalee, *The Order of Generalized Hypersubstitutions of Type  $\tau = (2)$* , International Journal of Mathematics and Mathematical Sciences, Vol 2008 (2008), Article ID 263541, 8 pages, doi:10.1155/2008/263541
- [7] W. Taylor, *Hyperidentities and Hypervarieties*, Aequationes Mathematicae **23** (1981), 111–127.
- [8] S.L. Wismath, *The monoid of hypersubstitutions of type  $(n)$* , Southeast Asian Bull. Math. **24** (1) (2000), 115–128.

Received 4 November 2009

Revised 17 November 2009