Discussiones Mathematicae
General Algebra and Applications 30 (2010) 173–191
doi:10.7151/dmgaa.1168

THE MONOID OF GENERALIZED HYPERSUBSTITUTIONS OF TYPE $\tau = (n)$

Wattapong Puninagool

and

Sorasak Leeratanavalee*

Department of Mathematics,
Faculty of Science, Chiang Mai University,
Chiang Mai 50200, Thailand

e-mail: wattapong1p@yahoo.com
e-mail: scislrtt@chiangmai.ac.th

Abstract

A (usual) hypersubstitution of type τ is a function which takes each operation symbol of the type to a term of the type, of the same arity. The set of all hypersubstitutions of a fixed type τ forms a monoid under composition, and semigroup properties of this monoid have been studied by a number of authors. In particular, idempotent and regular elements, and the Green's relations, have been studied for type (n) by S.L. Wismath.

A generalized hypersubstitution of type $\tau = (n)$ is a mapping σ which takes the n-ary operation symbol f to a term $\sigma(f)$ which does not necessarily preserve the arity. Any such σ can be inductively extended to a map $\hat{\sigma}$ on the set of all terms of type $\tau = (n)$, and any two such extensions can be composed in a natural way. Thus, the set $Hyp_G(n)$ of all generalized hypersubstitutions of type $\tau = (n)$ forms a monoid. In this paper we study the semigroup properties of $Hyp_G(n)$.

*Corresponding author.

In particular, we characterize the idempotent and regular generalized hypersubstitutions, and describe some classes under Green's relations of this monoid.

Keywords: monoid, regular elements, idempotent elements, Green's relations, generalized hypersubstitution.

2000 Mathematics Subject Classification: 20M05, 20M99, 20N02.

References

[1] K. Denecke, D. Lau, R. Pöschel and D. Schweigert, *Hyperidentities, Hyperequational Classes, and Clone Congruences*, Verlag Hölder-Pichler-Tempsky, Wien, Contributions to General Algebra **7** (1991), 97–118.

[2] S. Leeratanavalee and K. Denecke, *Generalized Hypersubstitutions and Strongly Solid Varieties*, p. 135–145 in: General Algebra and Applications, Proc. of the *"59 th Workshop on General Algebra", '15 th Conference for Young Algebraists Potsdam 2000'*, Shaker Verlag 2000.

[3] S. Leeratanavalee, *Submonoids of Generalized Hypersubstitutions*, Demonstratio Mathematica **XL** (1) (2007), 13–22.

[4] W. Puninagool and S. Leeratanavalee, *All Regular Elements in* $Hyp_G(2)$, preprint 2009.

[5] W. Puninagool and S. Leeratanavalee, *Green's Relations on* $Hyp_G(2)$, preprint 2009.

[6] W. Puninagool and S. Leeratanavalee, *The Order of Generalized Hypersubstitutions of Type* $\tau = (2)$, International Journal of Mathematics and Mathematical Sciences, Vol 2008 (2008), Article ID 263541, 8 pages, doi:10.1155/2008/263541

[7] W. Taylor, *Hyperidentities and Hypervarieties*, Aequationes Mathematicae **23** (1981), 111–127.

[8] S.L. Wismath, *The monoid of hypersubstitutions of type* (n), Southeast Asian Bull. Math. **24** (1) (2000), 115–128.

Received 4 November 2009
Revised 17 November 2009