

## SPECIAL M-HYPERIDENTITIES IN BIREGULAR LEFTMOST GRAPH VARIETIES OF TYPE (2,0)

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### Abstract

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph  $G$  satisfies a term equation  $s \approx t$  if the corresponding graph algebra  $A(G)$  satisfies  $s \approx t$ . A class of graph algebras  $V$  is called a graph variety  $\overline{V} = \text{Mod}_g \Sigma$  where  $\Sigma$  is a subset of  $T(X) \times T(X)$ . A graph variety  $V' = \text{Mod}_g \Sigma'$  is called a biregular leftmost graph variety if  $\Sigma'$  is a set of biregular leftmost term equations. A term equation  $s \approx t$  is called an identity in a variety  $V$  if  $A(G)$  satisfies  $s \approx t$  for all  $G \in V$ . An identity  $s \approx t$  of a variety  $V$  is called a hyperidentity of a graph algebra  $A(G)$ ,  $G \in V$  whenever the operation symbols occurring in  $s$  and  $t$  are replaced by any term operations of  $A(G)$  of the appropriate arity, the resulting identities hold in  $A(G)$ . An identity  $s \approx t$  of a variety  $V$  is called an  $M$ -hyperidentity of a graph algebra  $A(G)$ ,  $G \in V$  whenever the operation symbols occurring in  $s$  and  $t$  are replaced by any term operations in a subgroupoid  $M$  of term operations of  $A(G)$  of the appropriate arity, the resulting identities hold in  $A(G)$ .

In this paper we characterize special  $M$ -hyperidentities in each biregular leftmost graph variety. For identities, varieties and other basic concepts of universal algebra see e.g. [3].

**Keywords:** varieties, biregular leftmost graph varieties, identities, term, hyperidentity,  $M$ -hyperidentity, binary algebra, graph algebra.

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