

ON THE MATRIX NEGATIVE PELL EQUATION

ALEKSANDER GRYTCZUK AND IZABELA KURZYDŁO

Faculty of Mathematics, Computer Science and Econometrics
University of Zielona Góra
prof. Z. Szafrana 4a, 65-516 Zielona Góra, Poland

e-mail: A.Grytczuk@wmie.uz.zgora.pl

e-mail: I.Kurzydlo@wmie.uz.zgora.pl

Abstract

Let N be a set of natural numbers and Z be a set of integers. Let $M_2(Z)$ denotes the set of all 2×2 matrices with integer entries.

We give necessary and sufficient conditions for solvability of the matrix negative Pell equation

$$(P) \quad X^2 - dY^2 = -I \quad \text{with } d \in N$$

for nonsingular X, Y belonging to $M_2(Z)$ and his generalization

$$(Pn) \quad \sum_{i=1}^n X_i^2 - d \sum_{i=1}^n Y_i^2 = -I \quad \text{with } d \in N$$

for nonsingular $X_i, Y_i \in M_2(Z), i = 1, \dots, n$.

This paper is partly supported by EFS (European Social Funds).

Keywords: the matrix negative Pell equation, powers matrices.

2000 Mathematics Subject Classification: 15A24,15A42.

REFERENCES

- [1] Z. Cao and A. Grytczuk, *Fermat's type equations in the set of 2x2 integral matrices*, Tsukuba J. Math. **22** (1998), 637–643.
- [2] R.Z. Domiaty, Solutions of $x^4 + y^4 = z^4$ in 2x2 integral matrices, Amer. Math. Monthly (1966) **73**, 631.
- [3] A. Grytczuk, *Fermat's equation in the set of matrices and special functions*, Studia Univ. Babes-Bolyai, Mathematica **4** (1997), 49–55 .
- [4] A. Grytczuk, *On a conjecture about the equation $A^{mx} + A^{my} = A^{mz}$* , Acta Acad. Paed. Agriensis, Sectio Math. **25** (1998), 61–70.
- [5] A. Grytczuk and J. Grytczuk, *Ljunggren's trinomials and matrix equation $A^x + A^y = A^z$* , Tsukuba J. Math. **2** (2002), 229–235.
- [6] A. Grytczuk and K. Grytczuk, *Functional recurrences*, 115–121 in: Applications of Fibonacci Numbers, Ed. E. Bergum et als, by Kluwer Academic Publishers 1990.
- [7] A. Grytczuk, F. Luca and M. Wójcikowicz, *The negative Pell equation and Pythagorean triples*, Proc. Japan Acad. **76** (2000), 91–94.
- [8] A. Khazanov, *Fermat's equation in matrices*, Serdica Math. J. **21** (1995), 19–40.
- [9] I. Kurzydło, Explicit form on a GLW criterion for solvability of the negative Pell equation - Submitted.
- [10] M. Le and C. Li, *On Fermat's equation in integral 2x2 matrices*, Period. Math. Hung. **31** (1995), 219–222.
- [11] Z. Patay and A. Szakacs, *On Fermat's problem in matrix rings and groups*, Publ. Math. Debrecen **61** (3-4) (2002), 487–494.
- [12] H. Qin, *Fermat's problem and Goldbach problem over $M_n(Z)$* , Linear Algebra App., **236** (1996), 131–135.
- [13] P. Ribenboim, 13 Lectures on Fermat's Last Theorem (New York: Springer-Verlag) 1979.
- [14] N. Vaserstein, *Non-commutative Number Theory*, Contemp. Math. **83** (1989), 445–449.

Received 5 February 2009

Revised 10 March 2009