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ON THE LATTICE OF CONGRUENCES ON INVERSE SEMIRINGS

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Abstract

Let S be a semiring whose additive reduct $(S, +)$ is an inverse semigroup. The relations θ and k, induced by tr and ker (*resp.*), are congruences on the lattice $\mathcal{C}(S)$ of all congruences on S. For $\rho \in \mathcal{C}(S)$, we have introduced four congruences $\rho_{\min}$, $\rho_{\max}$, $\rho^{\min}$ and $\rho^{\max}$ on S and showed that $\rho\theta = [\rho_{\min}, \rho_{\max}]$ and $\rho\kappa = [\rho^{\min}, \rho^{\max}]$. Different properties of $\rho\theta$ and $\rho\kappa$ have been considered here. A congruence ρ on S is a Clifford congruence if and only if ρ_{max} is a distributive lattice congruence and ρ^{max} is a skew-ring congruence on S. If η (σ) is the least distributive lattice (resp. skew-ring) congruence on S then $\eta \cap \sigma$ is the least Clifford congruence on S.

Keywords: inverse semirings, trace, kernel, Clifford congruence, least Clifford congruence.

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