

k -NORMALIZATION AND $(k + 1)$ -LEVEL INFLATION OF VARIETIES

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Abstract

Let τ be a type of algebras. A common measurement of the complexity of terms of type τ is the depth of a term. For $k \geq 1$, an identity $s \approx t$ of type τ is said to be k -normal (with respect to this depth complexity measurement) if either $s = t$ or both s and t have depth $\geq k$. A variety is called k -normal if all its identities are k -normal. Taking $k = 1$ with respect to the usual depth valuation of terms gives the well-known property of normality of identities or varieties. For any variety V , there is a least k -normal variety $N_k(V)$ containing V , the variety determined by the set of all k -normal identities of V . The concept of k -normalization was introduced by K. Denecke and S.L. Wismath in [5], and an algebraic characterization of the elements of $N_k(V)$ in terms of the algebras in V was given in [4]. In [1] a simplified version of this characterization of $N_k(V)$ was given, in the special case of the 2-normalization of the variety V of all lattices, using a construction called the 3-level inflation of a lattice. In this paper we show that the analogous $(k + 1)$ -level inflation can be used to characterize the algebras of $N_k(V)$ for any variety V having a unary term which satisfies two technical conditions. This includes any variety V which satisfies $x \approx t(x)$ for some unary term t of depth at least k , and in particular any variety, such as the variety of lattices, which satisfies an idempotent identity.

Keywords: k -normal identities, k -normalization of a variety, $(k + 1)$ -level inflation of algebras.

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