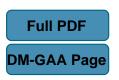
Discussiones Mathematicae General Algebra and Applications 28 (2008) 49–62 doi:10.7151/dmgaa.1134



k-NORMALIZATION AND (k+1)-LEVEL INFLATION OF VARIETIES

Valerie Cheng and Shelly Wismath

Math/C.S. Dept., University of Lethbridge
401 University Drive, Lethbridge, Ab., T1K-3M4 Canada

e-mail: wismaths@cs.uleth.ca

Abstract

Let τ be a type of algebras. A common measurement of the complexity of terms of type τ is the depth of a term. For $k \geq 1$, an identity $s \approx t$ of type τ is said to be k-normal (with respect to this depth complexity measurement) if either s = t or both s and t have depth $\geq k$. A variety is called k-normal if all its identities are k-normal. Taking k=1 with respect to the usual depth valuation of terms gives the wellknown property of normality of identities or varieties. For any variety V, there is a least k-normal variety $N_k(V)$ containing V, the variety determined by the set of all k-normal identities of V. The concept of k-normalization was introduced by K. Denecke and S.L. Wismath in [5], and an algebraic characterization of the elements of $N_k(V)$ in terms of the algebras in V was given in [4]. In [1] a simplified version of this characterization of $N_k(V)$ was given, in the special case of the 2-normalization of the variety V of all lattices, using a construction called the 3-level inflation of a lattice. In this paper we show that the analogous (k+1)-level inflation can be used to characterize the algebras of $N_k(V)$ for any variety V having a unary term which satisfies two technical conditions. This includes any variety V which satisfies $x \approx t(x)$ for some unary term t of depth at least k, and in particular any variety, such as the variety of lattices, which satisfies an idempotent identity.

Research supported by NSERC of Canada.

Keywords: k-normal identities, k-normalization of a variety, (k+1)-level inflation of algebras.

2000 Mathematics Subject Classification: 08A40, 08B15, 08B05.

References

- [1] I. Chajda, V. Cheng and S.L. Wismath, 2-Normalization of lattices, to appear in Czechoslovak Mathematics Journal.
- [2] A. Christie, Q. Wang and S.L. Wismath, *Minimal characteristic algebras for k-normality*, Scientiae Mathematicae Japonicae **61** (3) (2005), 547–565.
- [3] G.T. Clarke, Semigroup varieties of inflations of unions of groups, Semigroup Forum 23 (4) (1981), 311–319.
- [4] K. Denecke and S.L. Wismath, A characterization of k-normal varieties, Algebra Universalis **51** (4) (2004), 395–409.
- [5] K. Denecke and S.L. Wismath, *Valuations of terms*, Algebra Universalis **50** (1) (2003), 107–128.
- [6] E. Graczyńska, On normal and regular identities, Algebra Universalis 27 (3) (1990), 387–397.
- [7] I.I. Mel'nik, *Nilpotent shifts of varieties*, (in Russian), Mat. Zametki, **14** (1973), 703–712; English translation in: Math. Notes **14** (1973), 962–966.

Received 11 July 2006 Revised 30 May 2007