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k-NORMALIZATION AND $(k+1)$-LEVEL INFLATION OF VARIETIES

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Abstract

Let τ be a type of algebras. A common measurement of the complexity of terms of type τ is the depth of a term. For $k \geq 1$, an identity $s \approx t$ of type τ is said to be k-normal (with respect to this depth complexity measurement) if either $s = t$ or both s and t have depth $\geq k$. A variety is called k-normal if all its identities are k-normal. Taking $k = 1$ with respect to the usual depth valuation of terms gives the well-known property of normality of identities or varieties. For any variety V, there is a least k-normal variety $N_k(V)$ containing V, the variety determined by the set of all k-normal identities of V. The concept of k-normalization was introduced by K. Denecke and S.L. Wismath in [5], and an algebraic characterization of the elements of $N_k(V)$ in terms of the algebras in V was given in [4]. In [1] a simplified version of this characterization of $N_k(V)$ was given, in the special case of the 2-normalization of the variety V of all lattices, using a construction called the 3-level inflation of a lattice. In this paper we show that the analogous $(k+1)$-level inflation can be used to characterize the algebras of $N_k(V)$ for any variety V having a unary term which satisfies two technical conditions. This includes any variety V which satisfies $x \approx t(x)$ for some unary term t of depth at least k, and in particular any variety, such as the variety of lattices, which satisfies an idempotent identity.

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References

[1] I. Chajda, V. Cheng and S.L. Wismath, 2-*Normalization of lattices,* to appear in Czechoslovak Mathematics Journal.

[2] A. Christie, Q. Wang and S.L. Wismath, *Minimal characteristic algebras for k-normality,* Scientiae Mathematicae Japonicae **61** (3) (2005), 547–565.

[3] G.T. Clarke, *Semigroup varieties of inflations of unions of groups,* Semigroup Forum **23** (4) (1981), 311–319.

[4] K. Denecke and S.L. Wismath, *A characterization of k-normal varieties,* Algebra Universalis **51** (4) (2004), 395–409.

[5] K. Denecke and S.L. Wismath, *Valuations of terms,* Algebra Universalis **50** (1) (2003), 107–128.

[6] E. Graczyńska, *On normal and regular identities,* Algebra Universalis **27** (3) (1990), 387–397.

[7] I.I. Mel'nik, *Nilpotent shifts of varieties,* (in Russian), Mat. Zametki, **14** (1973), 703–712; English translation in: Math. Notes **14** (1973), 962–966.

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