

## INTERIOR AND CLOSURE OPERATORS ON BOUNDED COMMUTATIVE RESIDUATED $\ell$ -MONOIDS

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### Abstract

Topological Boolean algebras are generalizations of topological spaces defined by means of topological closure and interior operators, respectively. The authors in [14] generalized topological Boolean algebras to closure and interior operators of  $MV$ -algebras which are an algebraic counterpart of the Łukasiewicz infinite valued logic. In the paper, these kinds of operators are extended (and investigated) to the wide class of bounded commutative  $R\ell$ -monoids that contains e.g. the classes of  $BL$ -algebras (i.e., algebras of the Hájek's basic fuzzy logic) and Heyting algebras as proper subclasses.

**Keywords:** residuated  $\ell$ -monoid, residuated lattice, closure operator,  $BL$ -algebra,  $MV$ -algebra.

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## REFERENCES

- [1] P. Bahls, J. Cole, N. Galatos, P. Jipsen and C. Tsinakis, *Cancellative residuated lattices*, Alg. Univ. **50** (2003), 83–106.
- [2] K. Blount and C. Tsinakis, *The structure of residuated lattices*, Intern. J. Alg. Comp. **13** (2003), 437–461.
- [3] R.O.L. Cignoli, I.M.L. D'Ottaviano and D. Mundici, *Algebraic Foundations of Many-valued Reasoning*, Kluwer Acad. Publ., Dordrecht-Boston-London 2000.
- [4] A. Dvurečenskij and S. Pulmannová, *New Trends in Quantum Structures*, Kluwer Acad. Publ., Dordrecht-Boston-London 2000.
- [5] A. Dvurečenskij and J. Rachůnek, *Bounded commutative residuated  $l$ -monoids with general comparability and states*, Soft Comput. **10** (2006), 212–218.
- [6] P. Hájek, *Metamathematics of Fuzzy Logic*, Kluwer, Amsterdam 1998.
- [7] P. Jipsen and C. Tsinakis, *A survey of residuated lattices*, Ordered algebraic structures (ed. J. Martinez), Kluwer Acad. Publ. Dordrecht (2002), 19–56.
- [8] J. Kühr, *Dually Residuated Lattice Ordered Monoids*, Ph. D. Thesis, Palacký Univ., Olomouc 2003.
- [9] J. Rachůnek, *DRL-semigroups and MV-algebras*, Czechoslovak Math. J. **48** (1998), 365–372.
- [10] J. Rachůnek, *MV-algebras are categorically equivalent to a class of  $DRL_{1(i)}$ -semigroups*, Math. Bohemica **123** (1998), 437–441.
- [11] J. Rachůnek, *A duality between algebras of basic logic and bounded representable DRL-monoids*, Math. Bohemica **126** (2001), 561–569.
- [12] J. Rachůnek and V. Slezák, *Negation in bounded commutative DRL-monoids*, Czechoslovak Math. J. **56** (2006), 755–763.
- [13] J. Rachůnek and D. Šalounová, *Local bounded commutative residuated  $l$ -monoids*, Czechoslovak Math. J. **57** (2007), 395–406.
- [14] J. Rachůnek and F. Švrček, *MV-algebras with additive closure operators*, Acta Univ. Palacký, Mathematica **39** (2000), 183–189.
- [15] H. Rasiowa and R. Sikorski, *The Mathematics of Metamathematics*, Panstw. Wyd. Nauk., Warszawa 1963.
- [16] K.L.N. Swamy, *Dually residuated lattice ordered semigroups*, Math. Ann. **159** (1965), 105–114.

- [17] E. Turunen, Mathematics Behind Fuzzy Logic, Physica-Verlag, Heidelberg-New York 1999.

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