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PRIME IDEAL THEOREM FOR DOUBLE BOOLEAN ALGEBRAS

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To the memory of Professor Kazimierz Głazek

Abstract

Double Boolean algebras are algebras $(D, \sqcap, \sqcup, ^{\lhd}, ^{\rhd}, \bot, \top)$ of type $(2, 2, 1, 1, 0, 0)$. They have been introduced to capture the equational theory of the algebra of protoconcepts. A filter (resp. an ideal) of a double Boolean algebra D is an upper set F (resp. down set I) closed under $\sqcap$ (resp. $\sqcup$). A filter F is called primary if $F \neq \emptyset$ and for all $x \in D$ we have $x \in F$ or $x^{\lhd} \in F$. In this note we prove that if F is a filter and I an ideal such that $F \cap I = \emptyset$ then there is a primary filter G containing F such that $G \cap I = \emptyset$ (i.e. the Prime Ideal Theorem for double Boolean algebras).

Keywords: double Boolean algebra, protoconcept algebra, concept algebra, weakly dicomplemented lattices.

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