Discussiones Mathematicae
General Algebra and Applications 27 (2007) 245–262
doi:10.7151/dmgaa.1129

Nd-SOLID VARIETIES

Klaus Denecke

Universität Potsdam, Fachbereich Mathematik
Postfach 601553, 14415 Potsdam, Germany
e-mail: kdenecke@rz.uni-potsdam.de

and

Prisana Glubudom

Chiangmai University, Department of Mathematics
Chiangmai, Thailand 50200
e-mail: puprisana@yahoo.com

To the memory of Professor Kazimierz Głazek

Abstract

A non-deterministic hypersubstitution maps any operation symbol of a tree language of type τ to a set of trees of the same type, i.e. to a tree language. Non-deterministic hypersubstitutions can be extended to mappings which map tree languages to tree languages preserving the arities. We define the application of a non-deterministic hypersubstitution to an algebra of type τ and obtain a class of derived algebras. Non-deterministic hypersubstitutions can also be applied to equations of type τ. Formally, we obtain two closure operators which turn out to form a conjugate pair of completely additive closure operators. This allows us to use the theory of conjugate pairs of additive closure operators for a characterization of M-solid non-deterministic varieties of algebras. As an application we consider M-solid non-deterministic varieties of semigroups.

Keywords: Non-deterministic hypersubstitution, conjugate pair of additive closure operators, M-solid non-deterministic variety.

2000 Mathematics Subject Classification: 08A35, 08A40, 08A70.

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Received 30 May 2006
Revised 15 June 2007